# Last time: Problem-Solving

- **Problem solving:**
  - Goal formulation
  - Problem formulation (states, operators)
  - Search for solution

- **Problem formulation:**
  - Initial state
  - ?
  - ?
  - ?

- **Problem types:**
  - single state: accessible and deterministic environment
  - multiple state: ?
  - contingency: ?
  - exploration: ?

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- **Problem solving:**
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  - Initial state
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  - Goal test
  - Path cost

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  - Goal formulation
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- **Problem formulation:**
  - Initial state
  - Operators
  - Goal test
  - Path cost

- **Problem types:**
  - single state: accessible and deterministic environment
  - multiple state: inaccessible and deterministic environment
  - contingency: inaccessible and nondeterministic environment
  - exploration: unknown state-space

# Last time: Finding a solution

**Solution:** is ???

**Basic idea:** offline, systematic exploration of simulated state-space by generating successors of explored states (expanding)

```
Function General-Search(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add resulting nodes to the search tree
    end
```

# Last time: Finding a solution

**Solution:** is a sequence of operators that bring you from current state to the goal state.

**Basic idea:** offline, systematic exploration of simulated state-space by generating successors of explored states (expanding).

**Function** General-Search(*problem*, *strategy*) returns a *solution*, or failure
    initialize the search tree using the initial state problem
    **loop do**
        **if** there are no candidates for expansion **then return** failure
        choose a leaf node for expansion according to strategy
        **if** the node contains a goal state **then return** the corresponding solution
        **else** expand the node and add resulting nodes to the search tree
    **end**

**Strategy:** The search strategy is determined by ???

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        if the node contains a goal state then return the corresponding solution
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    end
```

**Strategy:** The search strategy is determined by the **order** in which the nodes are expanded.

# A Clean Robust Algorithm

```
Function UniformCost-Search(problem, Queuing-Fn) returns a solution, or failure
    open ← make-queue(make-node(initial-state[problem]))
    closed ← [empty]
    loop do
        if open is empty then return failure
        currnode ← Remove-Front(open)
        if Goal-Test[problem] applied to State(currnode) then return currnode
        children ← Expand(currnode, Operators[problem])
        while children not empty

                    [... see next slide ...]
        end
        closed ← Insert(closed, currnode)
        open ← Sort-By-PathCost(open)
    end
```

## A Clean Robust Algorithm

```
[... see previous slide ...]
    children ← Expand(currnode, Operators[problem])
    while children not empty
        child ← Remove-Front(children)
        if no node in open or closed has child's state
            open ← Queuing-Fn(open, child)
        else if there exists node in open that has child's state
            if PathCost(child) < PathCost(node)
                open ← Delete-Node(open, node)
                open ← Queuing-Fn(open, child)
        else if there exists node in closed that has child's state
            if PathCost(child) < PathCost(node)
                closed ← Delete-Node(closed, node)
                open ← Queuing-Fn(open, child)
    end
[... see previous slide ...]
```

# Last time: search strategies

**Uninformed:** Use only information available in the problem formulation

- Breadth-first
- Uniform-cost
- Depth-first
- Depth-limited
- Iterative deepening

Informed: Use heuristics to guide the search

- Best first
- A*

# Evaluation of search strategies

- Search algorithms are commonly evaluated according to the following four criteria:
  - **Completeness:** does it always find a solution if one exists?
  - **Time complexity:** how long does it take as a function of number of nodes?
  - **Space complexity:** how much memory does it require?
  - **Optimality:** does it guarantee the least-cost solution?

- Time and space complexity are measured in terms of:
  - $b$ – max branching factor of the search tree
  - $d$ – depth of the least-cost solution
  - $m$ – max depth of the search tree (may be infinity)

# Last time: uninformed search strategies

**Uninformed search:**

Use only information available in the problem formulation

- Breadth-first
- Uniform-cost
- Depth-first
- Depth-limited
- Iterative deepening

# This time: informed search

**Informed search:**

Use heuristics to guide the search

- Best first
- A*
- Heuristics
- Hill-climbing
- Simulated annealing

# Best-first search

- Idea:

  use an evaluation function for each node; estimate of ***"desirability"***

$\Rightarrow$ expand most desirable unexpanded node.

- Implementation:

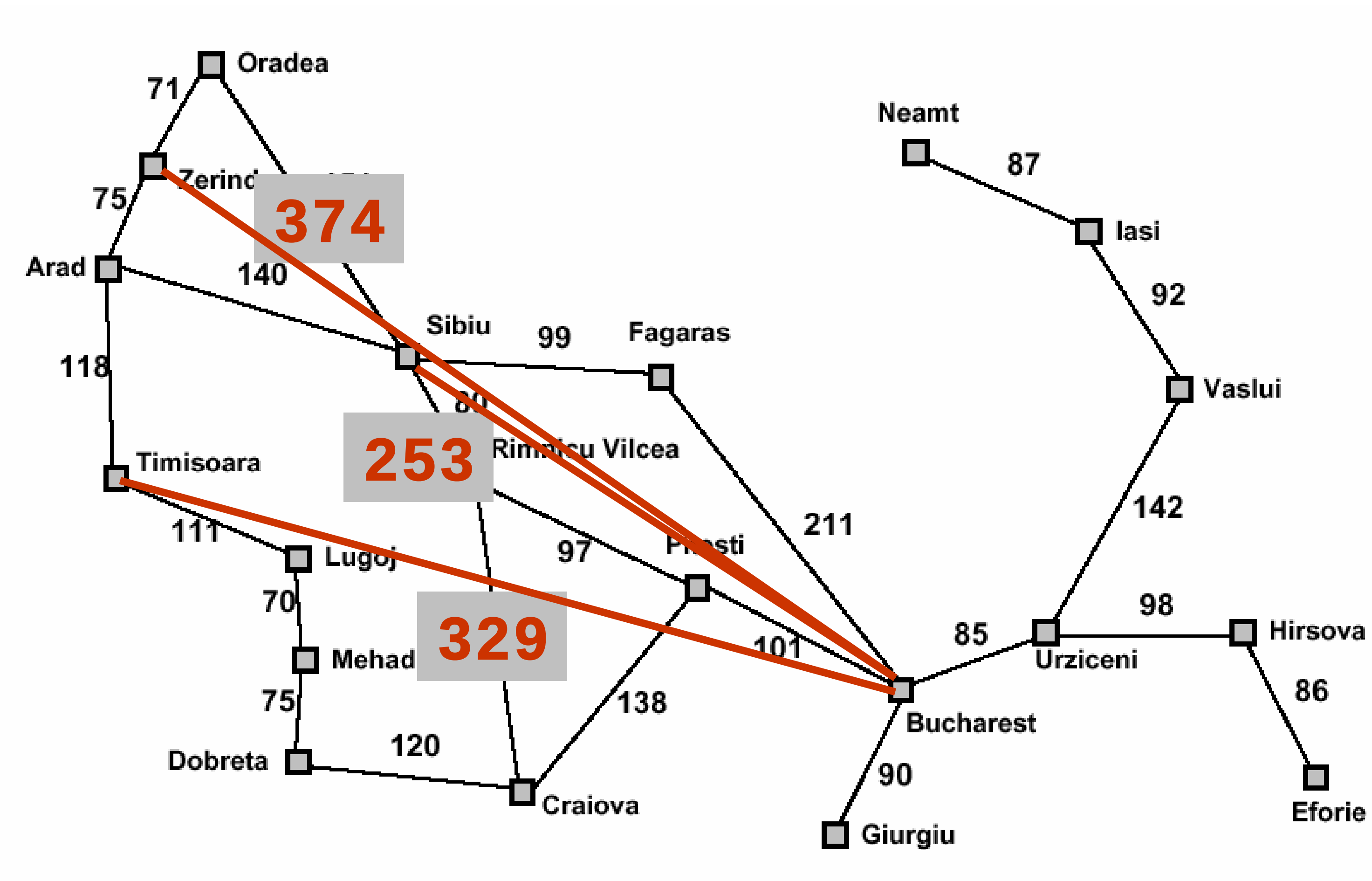
  **QueueingFn** = insert successors in decreasing order of desirability

- Special cases:

  greedy search

  A* search

# Romania with step costs in km




Straight–line distance to Bucharest

| | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Eforie** | 161 |
| **Fagaras** | 178 |
| **Giurgiu** | 77 |
| **Hirsova** | 151 |
| **Iasi** | 226 |
| **Lugoj** | 244 |
| **Mehadia** | 241 |
| **Neamt** | 234 |
| **Oradea** | 380 |
| **Pitesti** | 98 |
| **Rimnicu Vilcea** | 193 |
| **Sibiu** | 253 |
| **Timisoara** | 329 |
| **Urziceni** | 80 |
| **Vaslui** | 199 |
| **Zerind** | 374 |

## Greedy search

- Estimation function:

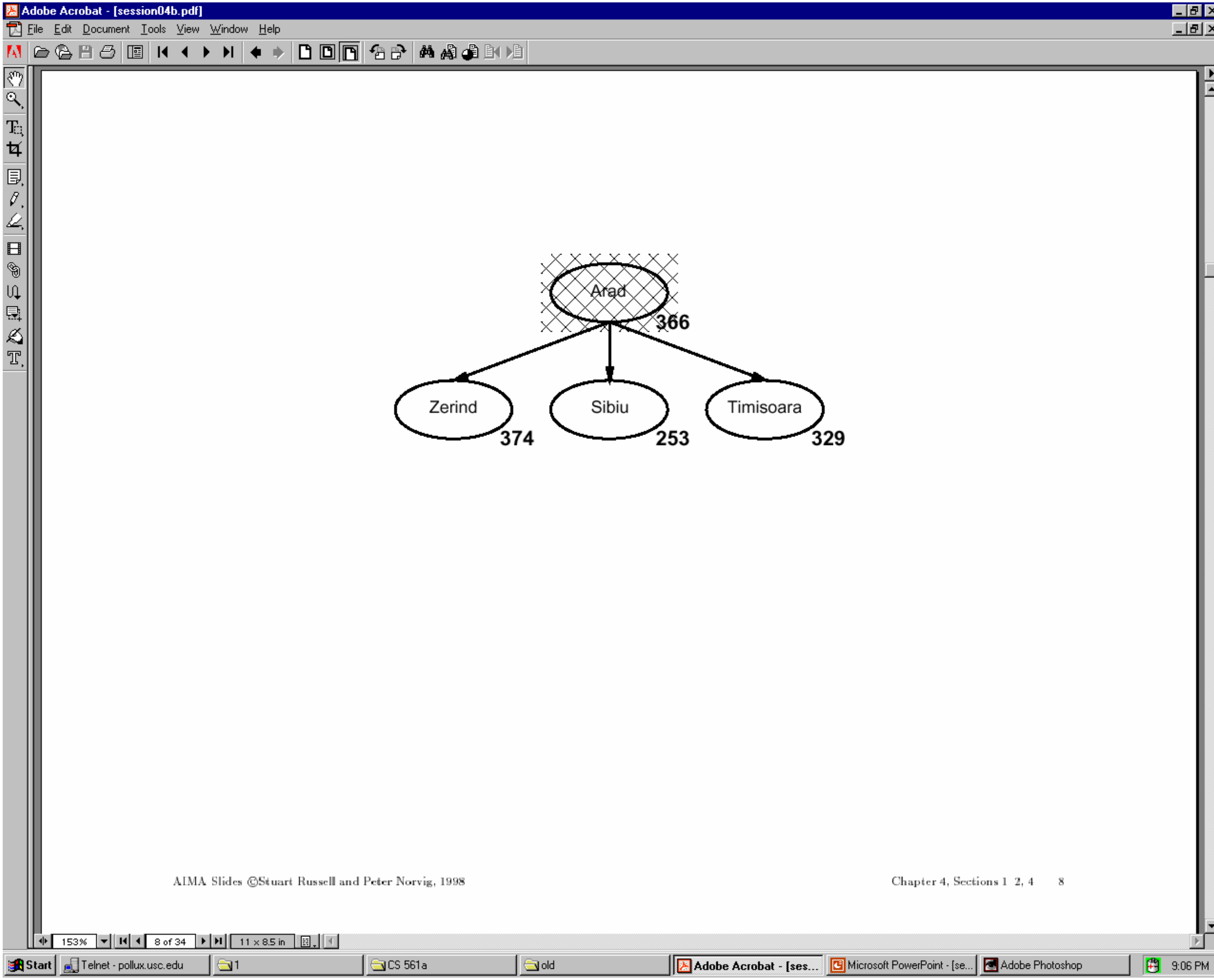
  *h(n)* = estimate of cost from *n* to goal (heuristic)

- For example:

  $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

- Greedy search expands first the node that appears to be closest to the goal, according to *h(n).*

# Greedy search example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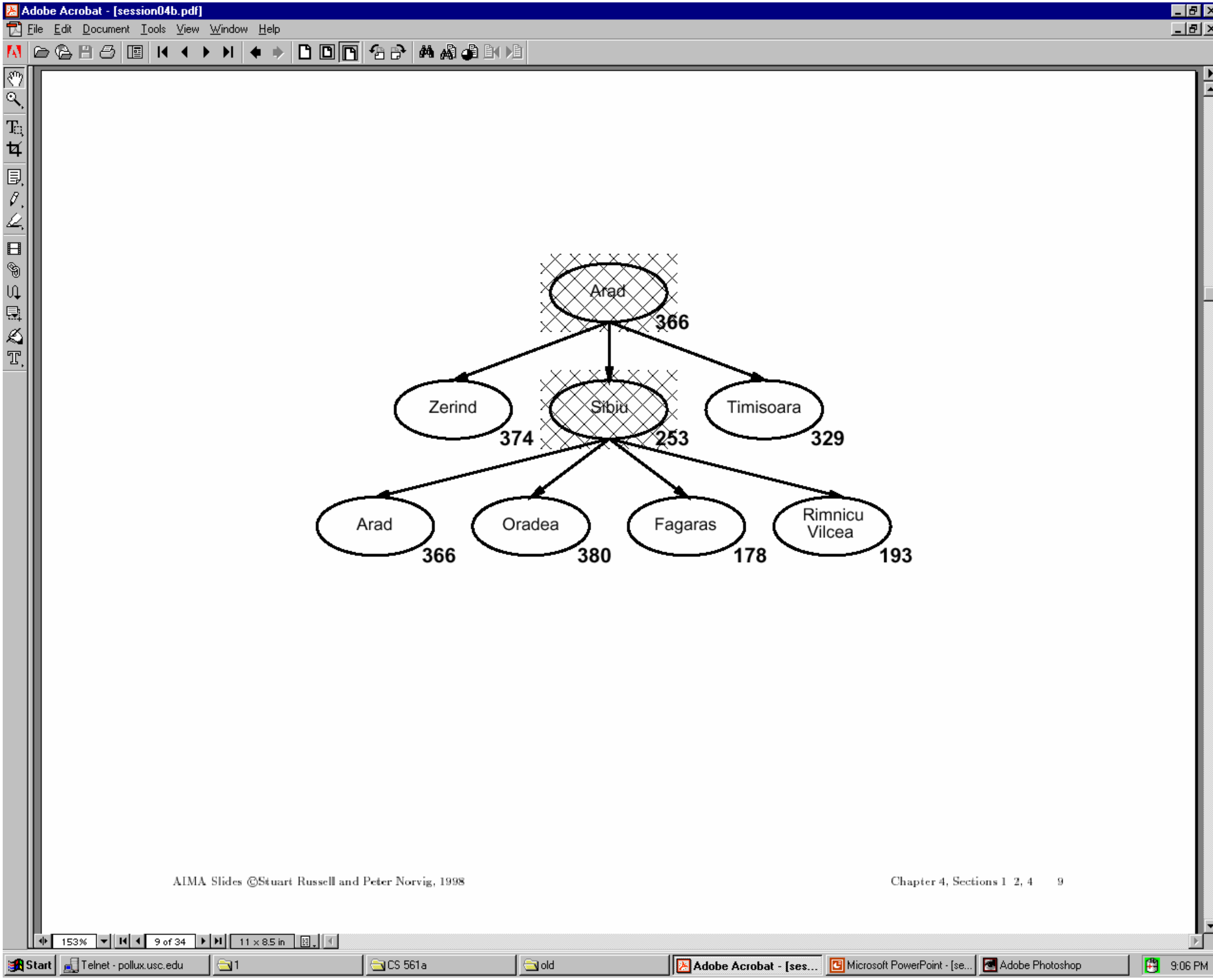
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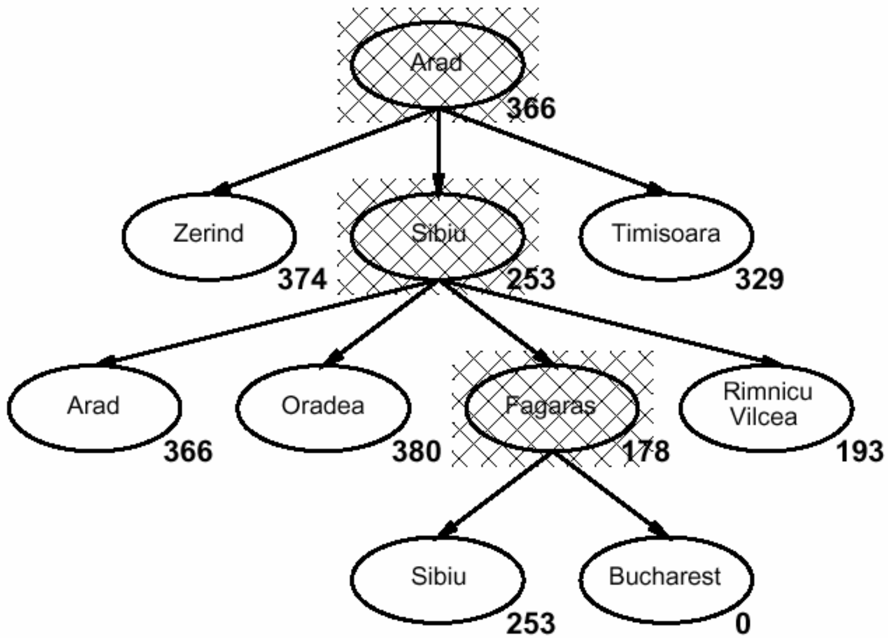




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## Properties of Greedy Search

- Complete?

- Time?

- Space?

- Optimal?

# Properties of Greedy Search

- Complete? No – can get stuck in loops
  e.g., Iasi > Neamt > Iasi > Neamt > ...
  Complete in finite space with repeated-state checking.
- Time? O(b^m) but a good heuristic can give dramatic improvement
- Space? O(b^m) – keeps all nodes in memory
- Optimal? No.

# A* search

- Idea: avoid expanding paths that are already expensive

  evaluation function: $f(n) = g(n) + h(n)$ with:

  $g(n)$ – cost so far to reach $n$

  $h(n)$ – estimated cost to goal from $n$

  $f(n)$ – estimated total cost of path through $n$ to goal

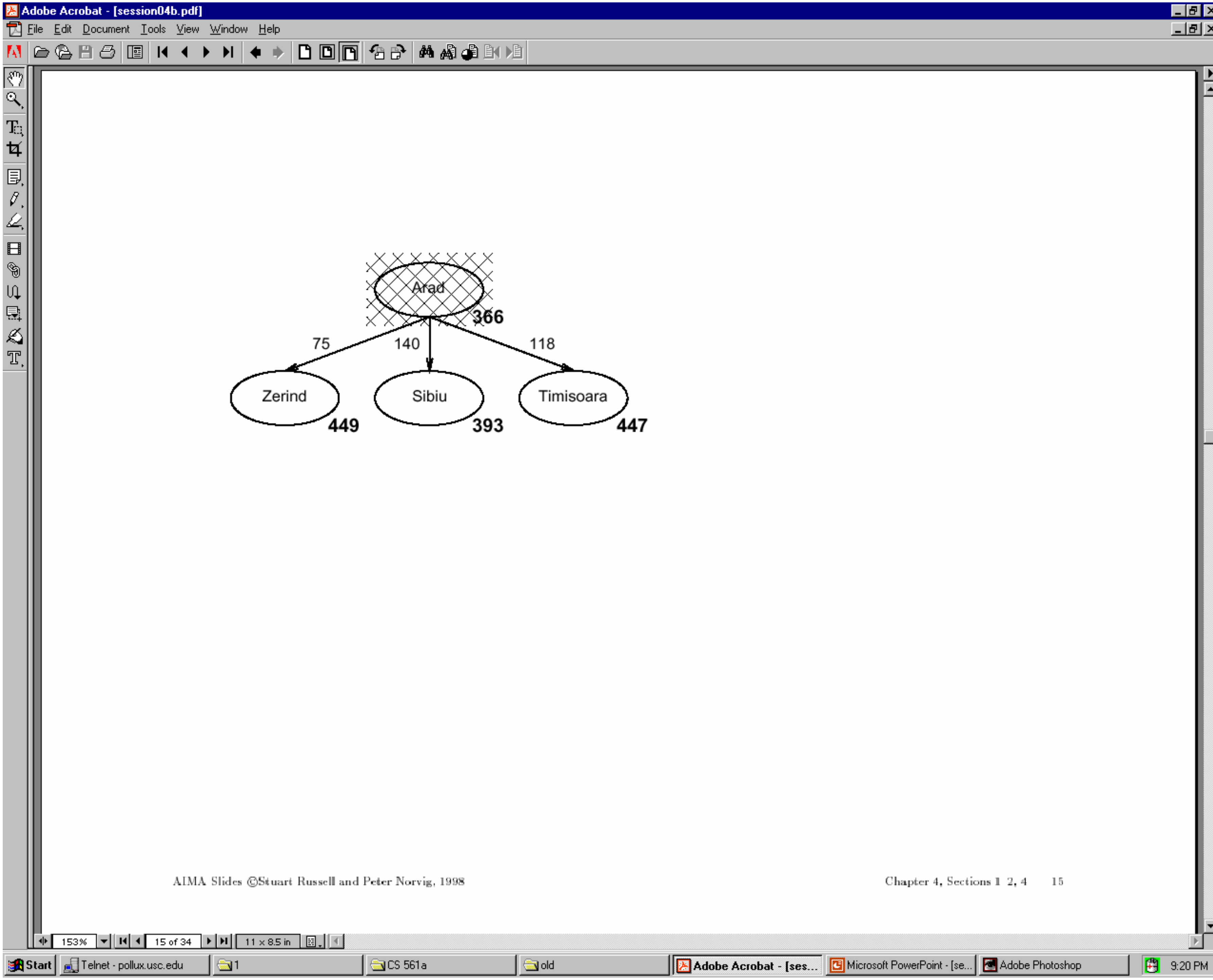
- A* search uses an admissible heuristic, that is,

  $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from $n$.

  For example: $h_{SLD}(n)$ never overestimates actual road distance.

- Theorem: A* search is optimal

# $A^*$ search example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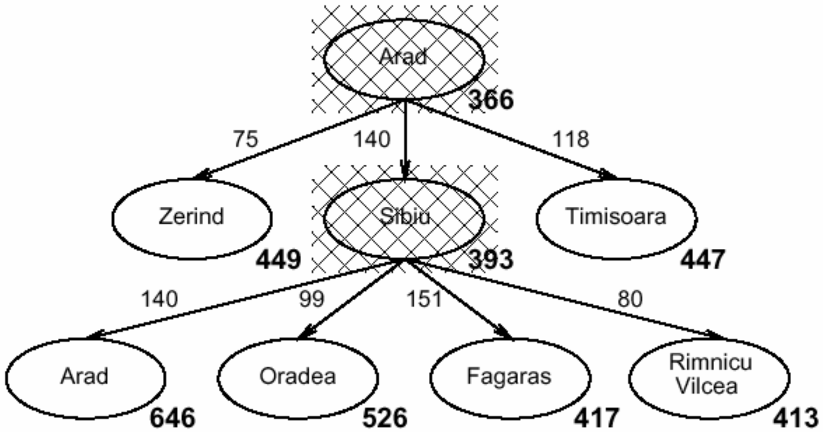
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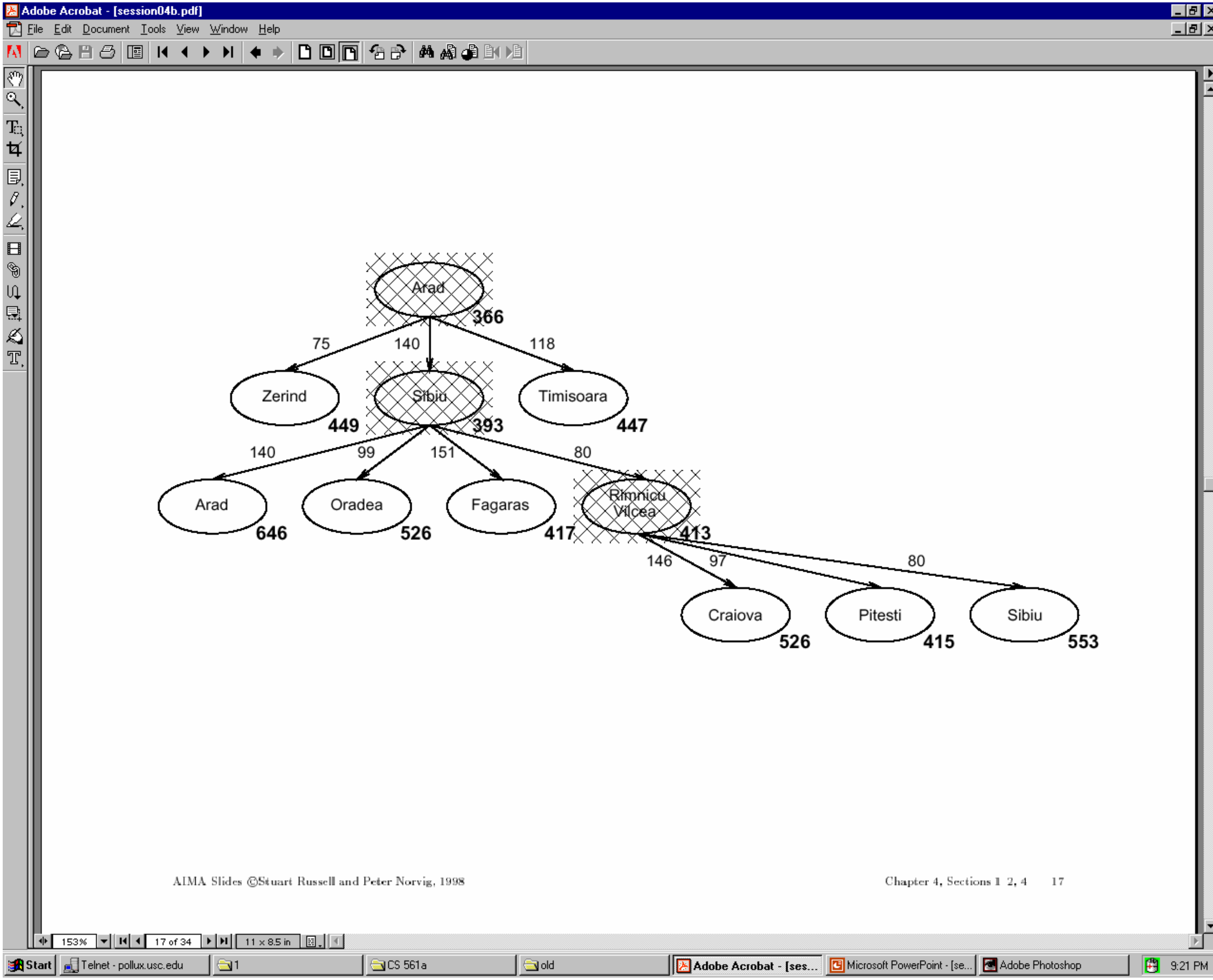
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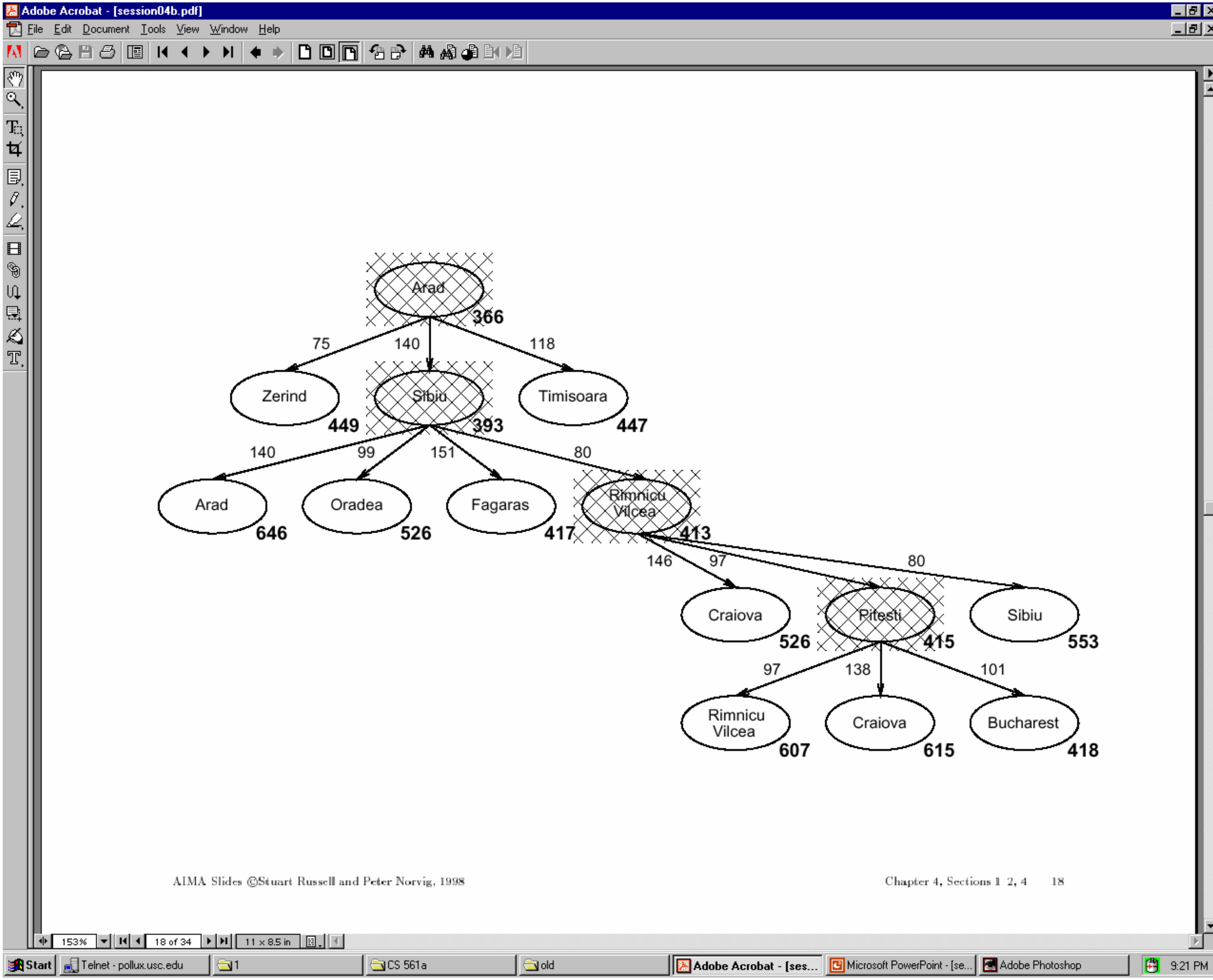
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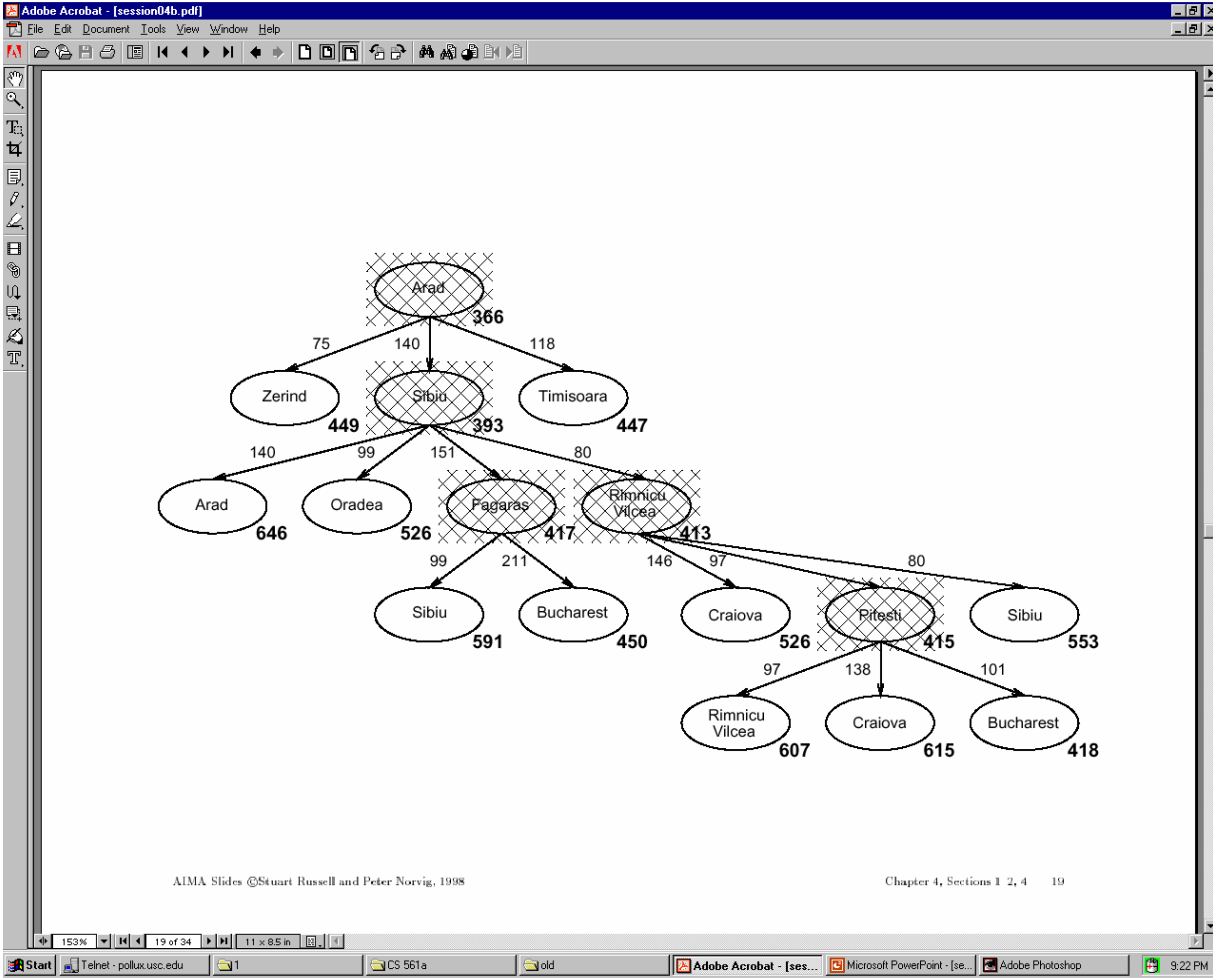
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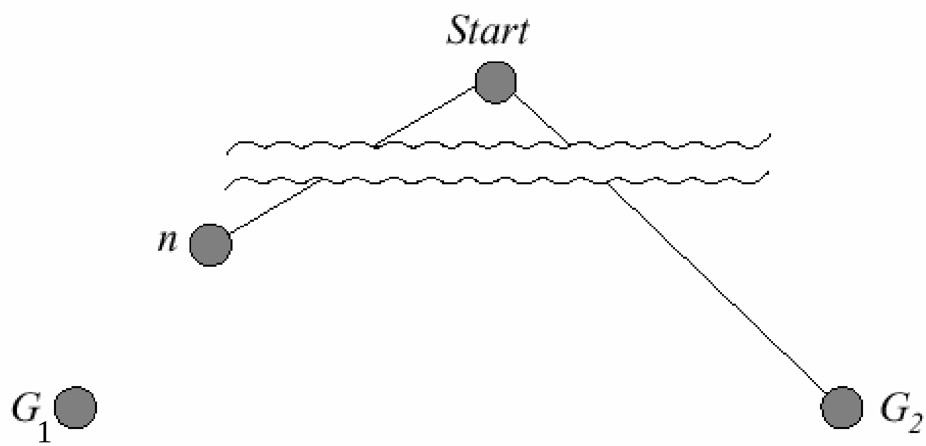
# Optimality of A* (standard proof)

Suppose some suboptimal goal $G_2$ has been generated and is in the queue. Let $n$ be an unexpanded node on a shortest path to an optimal goal $G_1$.



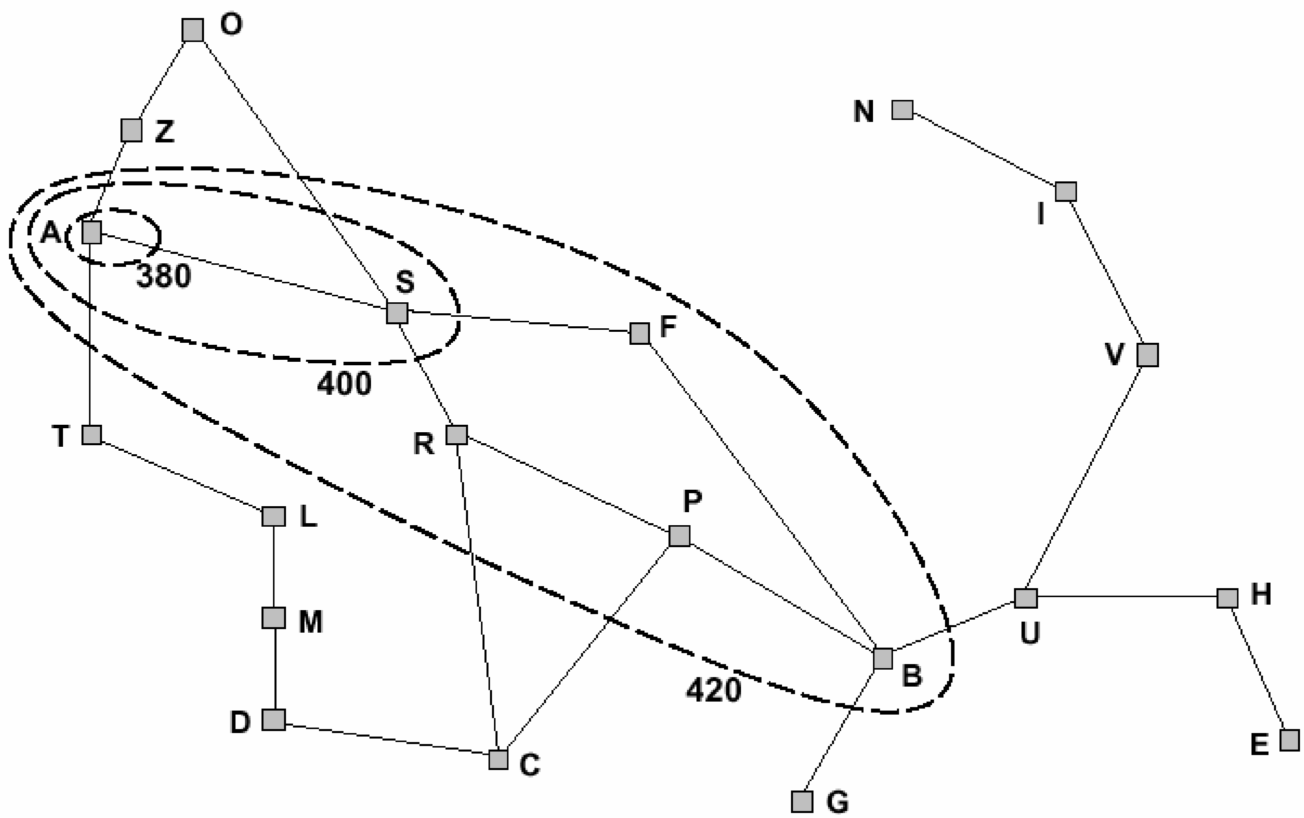

$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select $G_2$ for expansion

# Optimality of A* (more useful proof)

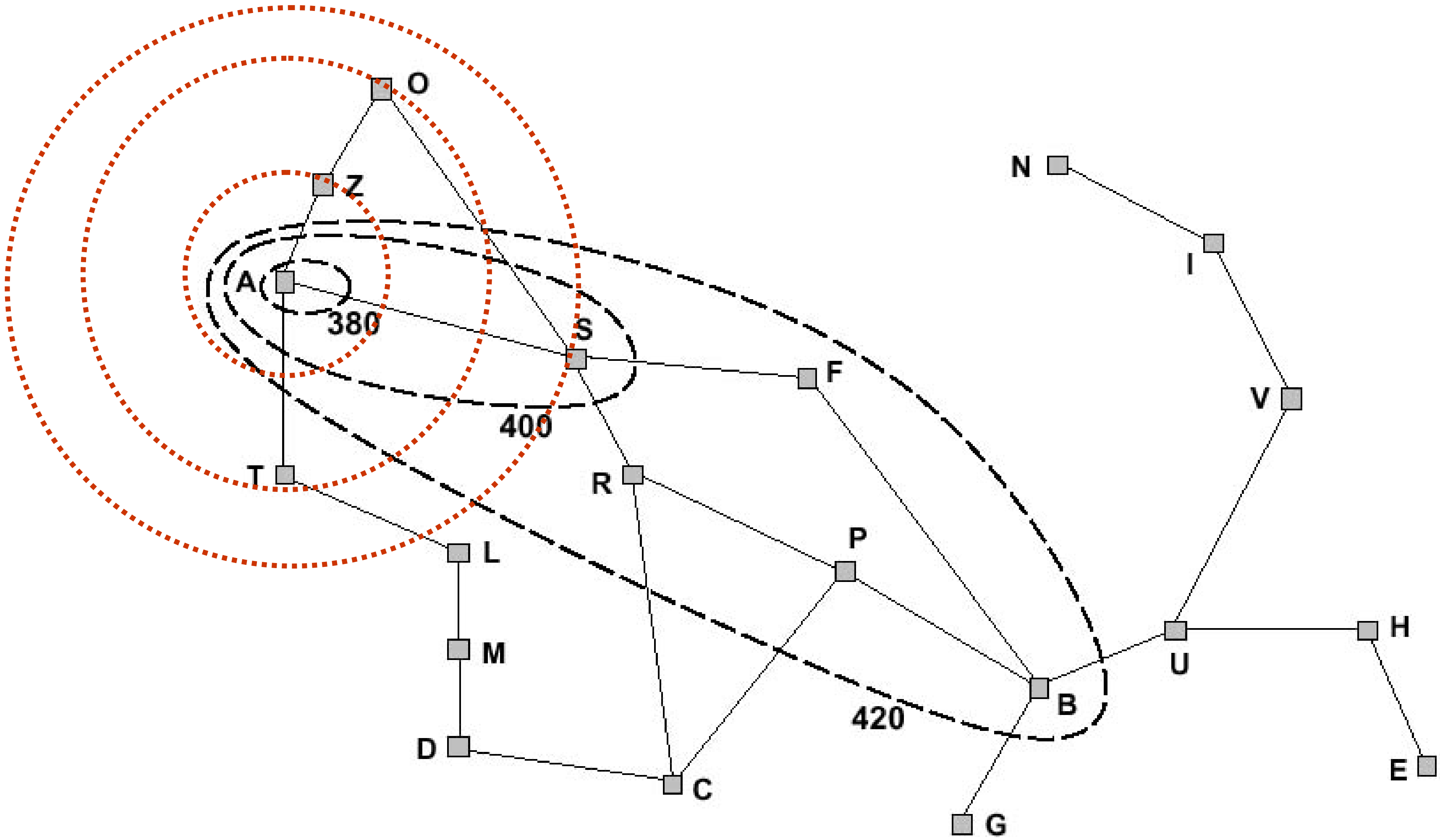
Lemma: A$^*$ expands nodes in order of increasing $f$ value

Gradually adds "$f$-contours" of nodes (cf. breadth-first adds layers)
Contour $i$ has all nodes with $f = f_i$, where $f_i < f_{i+1}$



## f-contours

How do the contours look like when h(n) =0?



# Properties of A*

- Complete?
- Time?
- Space?
- Optimal?

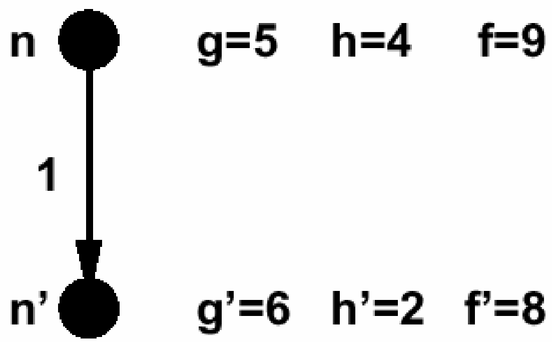
# Properties of A*

- Complete? Yes, unless infinitely many nodes with $f \leq f(G)$
- Time? Exponential in [(relative error in h) x (length of solution)]
- Space? Keeps all nodes in memory
- Optimal? Yes – cannot expand $f_{i+1}$ until $f_i$ is finished

# Proof of lemma: pathmax

For some admissible heuristics, $f$ may *decrease* along a path

E.g., suppose $n'$ is a successor of $n$




But this throws away information!
$f(n) = 9 \Rightarrow$ true cost of a path through $n$ is $\geq 9$
Hence true cost of a path through $n'$ is $\geq 9$ also

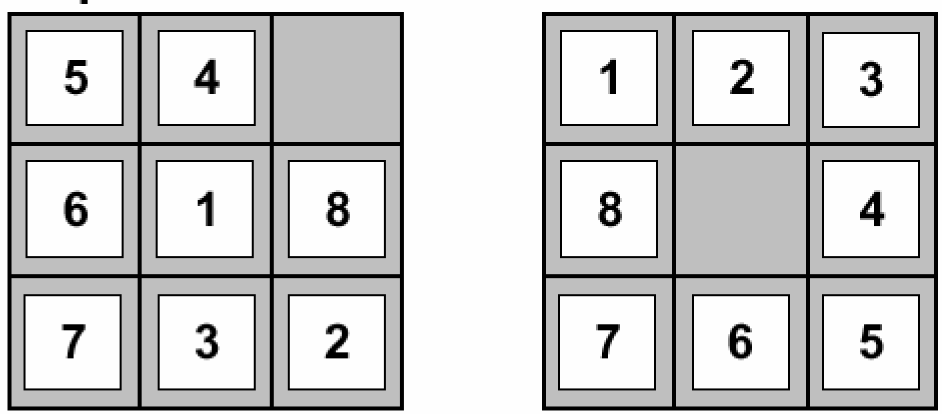
Pathmax modification to A$^*$:
Instead of $f(n') = g(n') + h(n')$, use $f(n') = max(g(n') + h(n'), f(n))$

With pathmax, $f$ is always nondecreasing along any path

# Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)




Start State    Goal State

$h_1(S)$ =??
$h_2(S)$ =??

# Admissible heuristics

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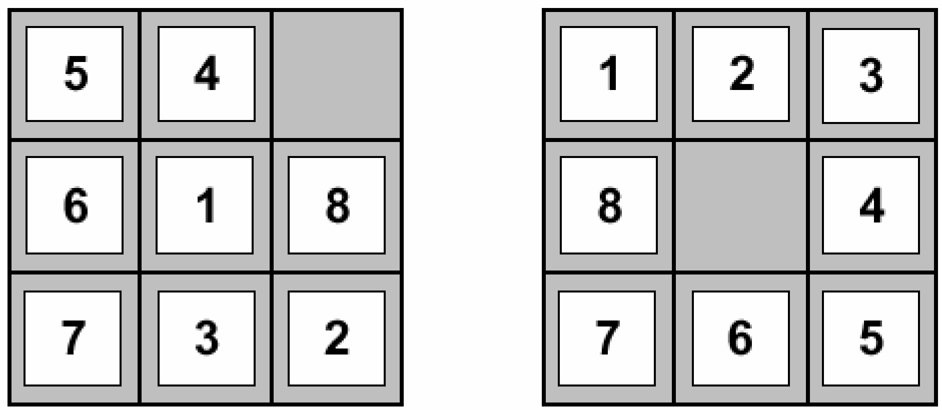


Start State Goal State

$h_1(S)$ =?? 7
$h_2(S)$ =?? 2+3+3+2+4+2+0+2 = 18

## Relaxed Problem

- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem.
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution.

## Next time

- Iterative improvement
- Hill climbing
- Simulated annealing