

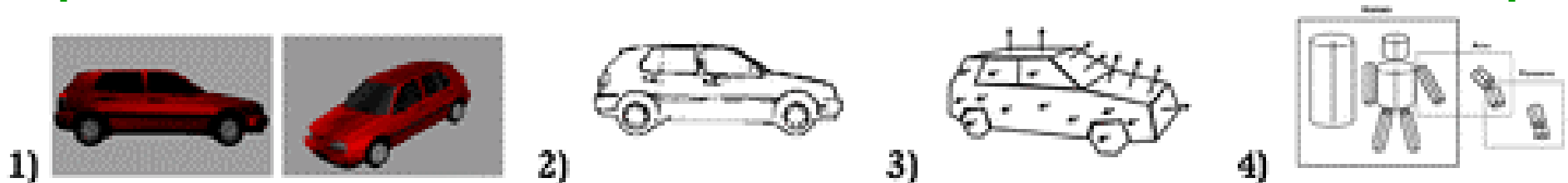
Lecture 18: Visual Pre-Processing.

Reading Assignments:

Chapters TMB2 3.3.

Low-Level Processing

Remember: Vision as a change in representation.



At the low-level, such change can be done by fairly streamlined mathematical transforms:

- Fourier transform
- **Wavelet transform**

these transforms yield a simpler but more organized image of the input.

Additional organization is obtained through **multiscale representations**.

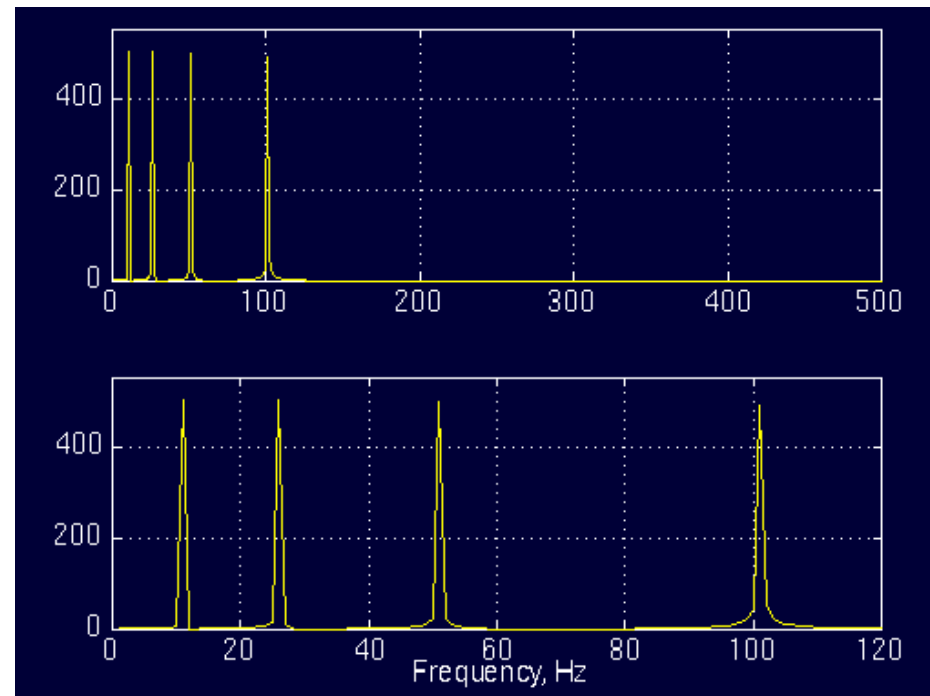
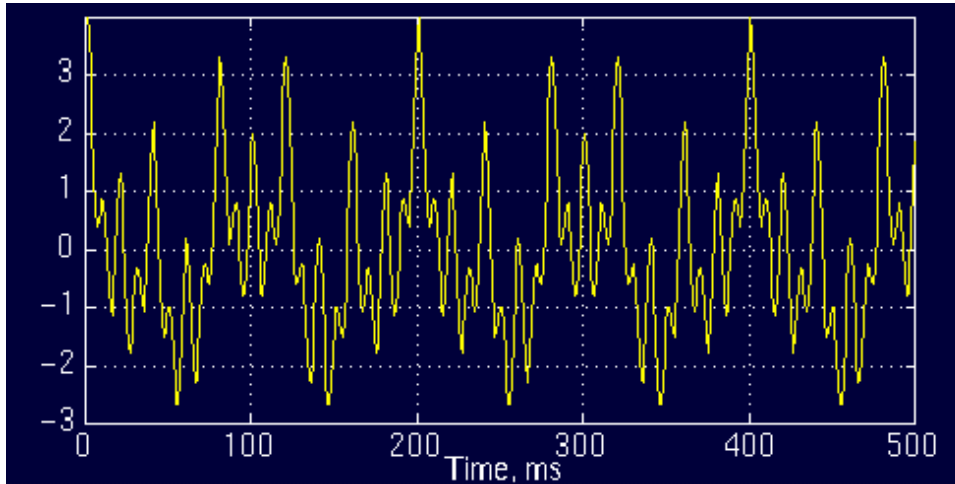
Biological low-level processing

Edge detection and wavelet transforms in V1: **hypercolumns** and **Jets**.

but...

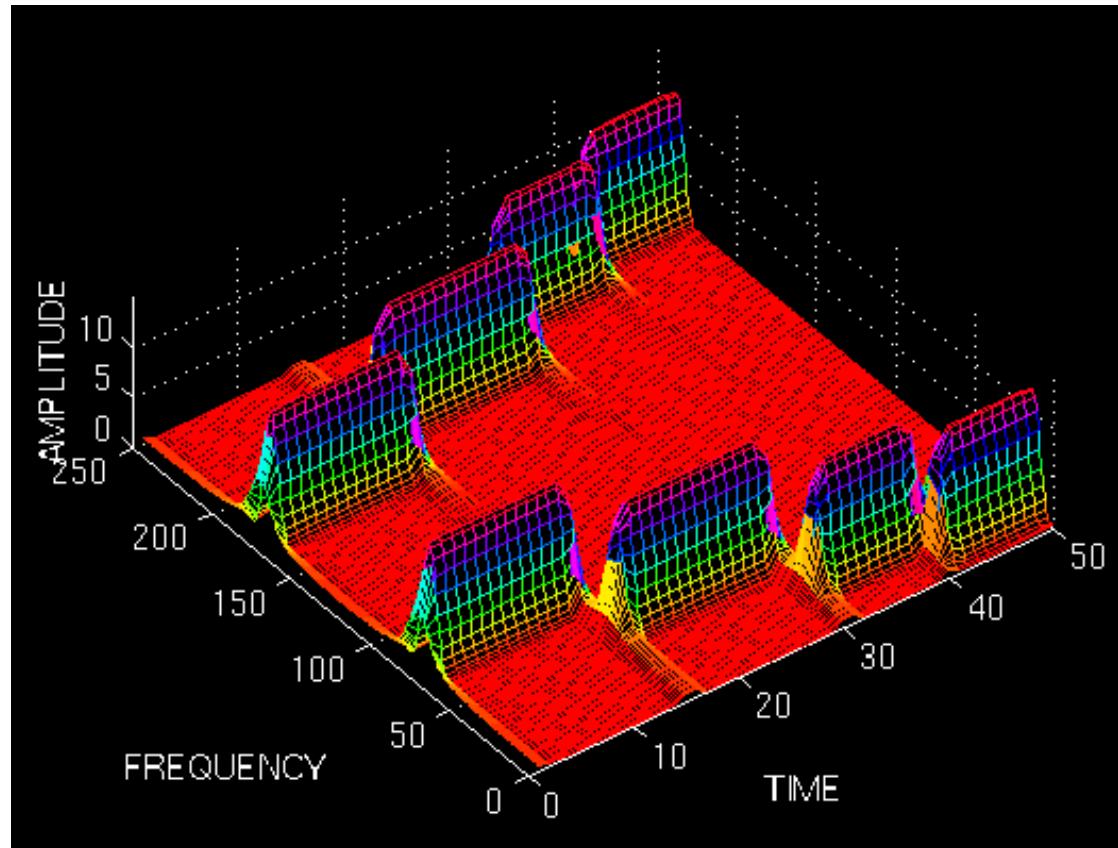
- processing appears highly **non-linear**, hence convolution of input by wavelets only approximates real responses;
- neuronal responses are influenced by **context**, i.e., neuronal activity at one location depends on activity at possibly distant locations;
- responses at one level of processing (e.g., V1) also depend on **feedback** from higher levels, and other modulatory effects such as attention, training, etc.

Fourier Transform



Problem

- The Fourier transform does not intuitively encode non-stationary (i.e., time-varying) signals.
- One solution is to use the short-term Fourier transform, and repeat for successive time slices.
- Another is to use a wavelet transform.

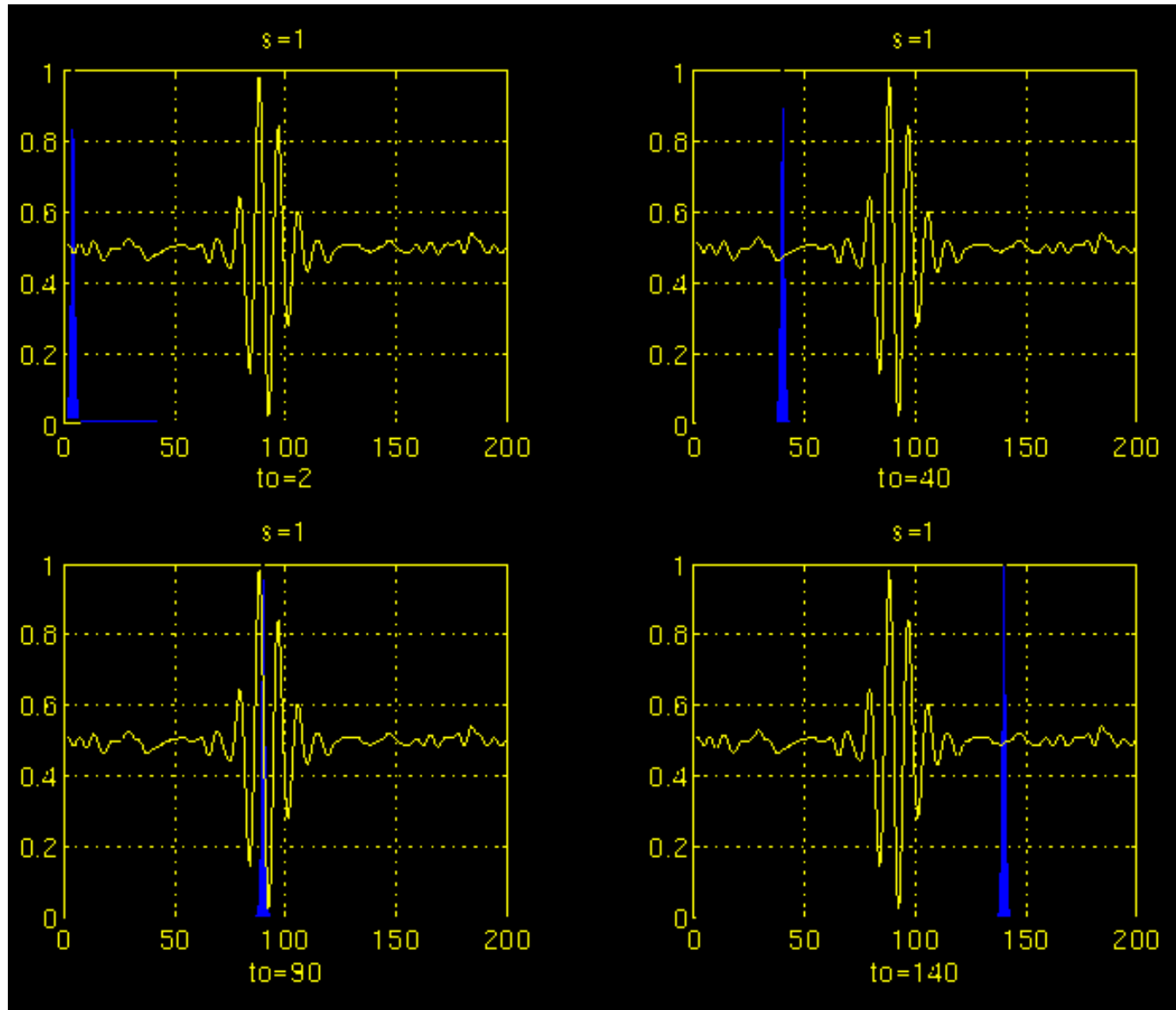


Wavelet Transform

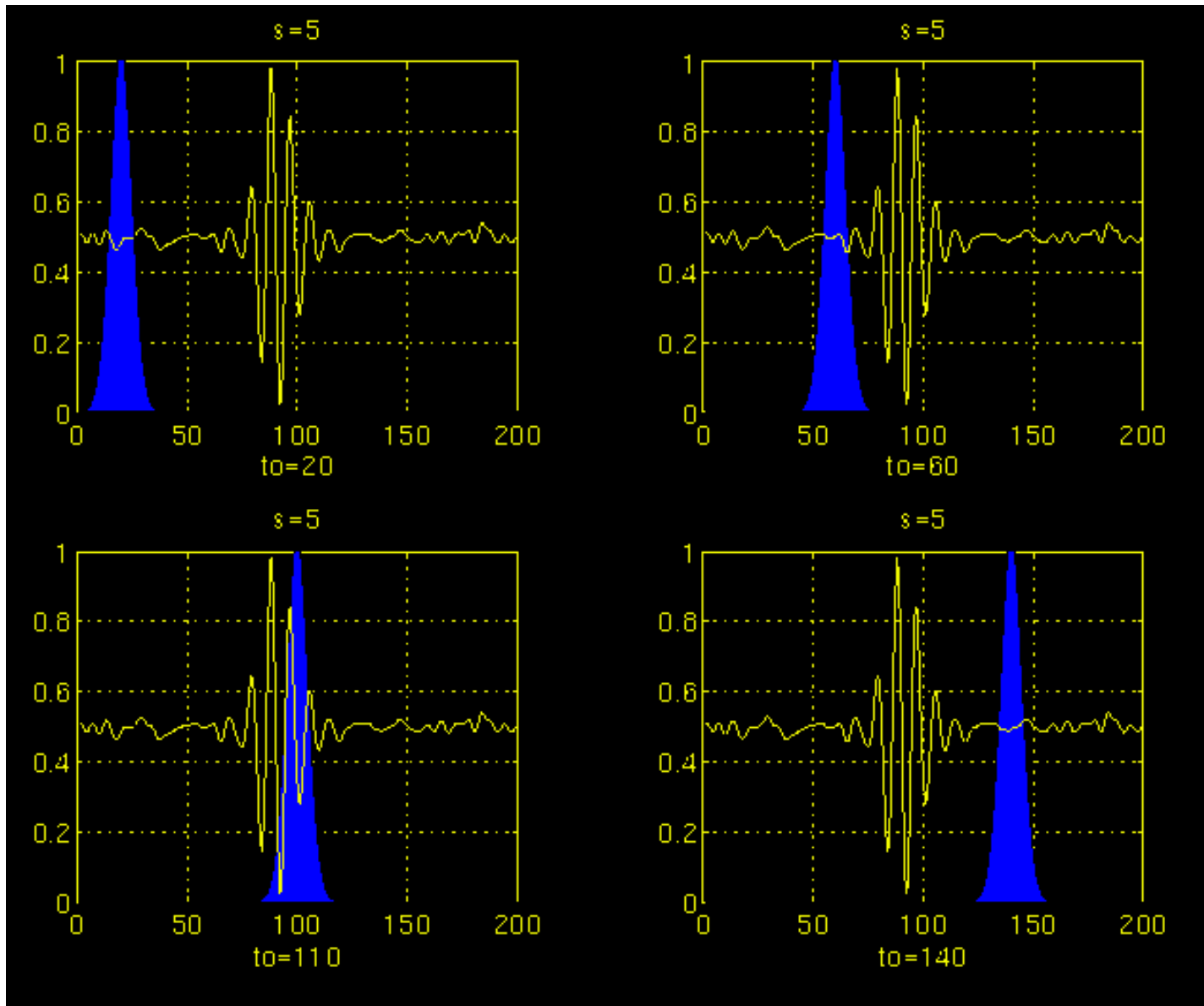
- Mother wavelet ψ : defines shape and size of window
- Convolved with signal (x) after translation (τ) and scaling (s)
- Results stored in array indexed by translation and scaling

$$CWT_x^\psi(\tau, s) = \Psi_x^\psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left(\frac{t - \tau}{s} \right) dt$$

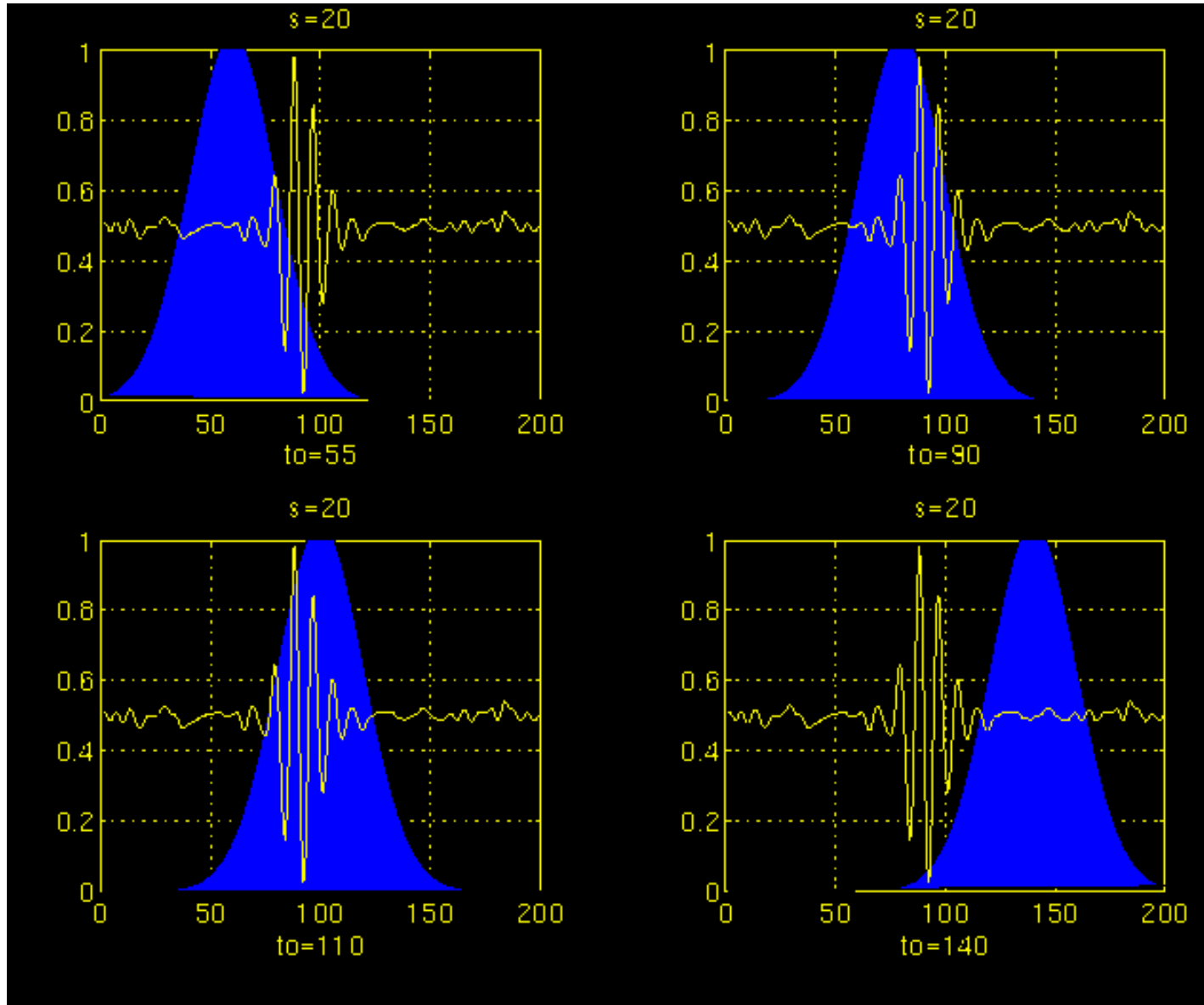
Example: small-scale wavelet is applied



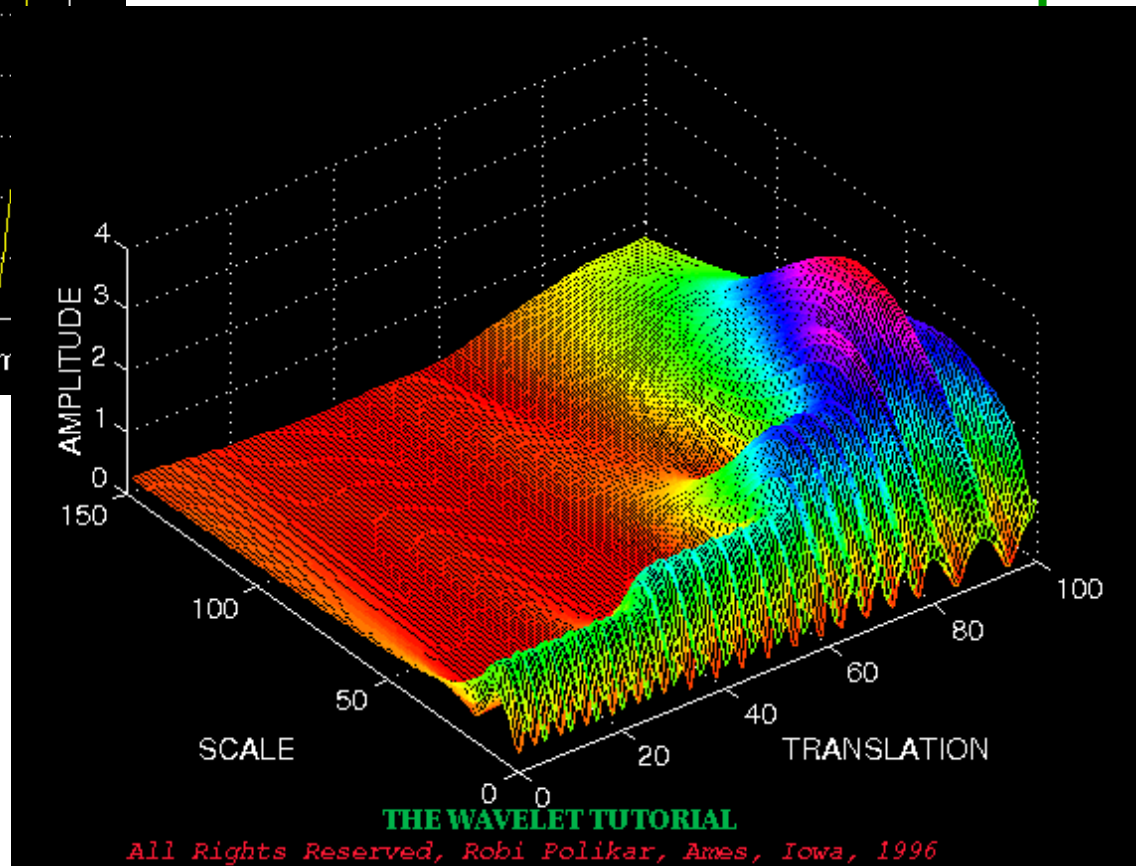
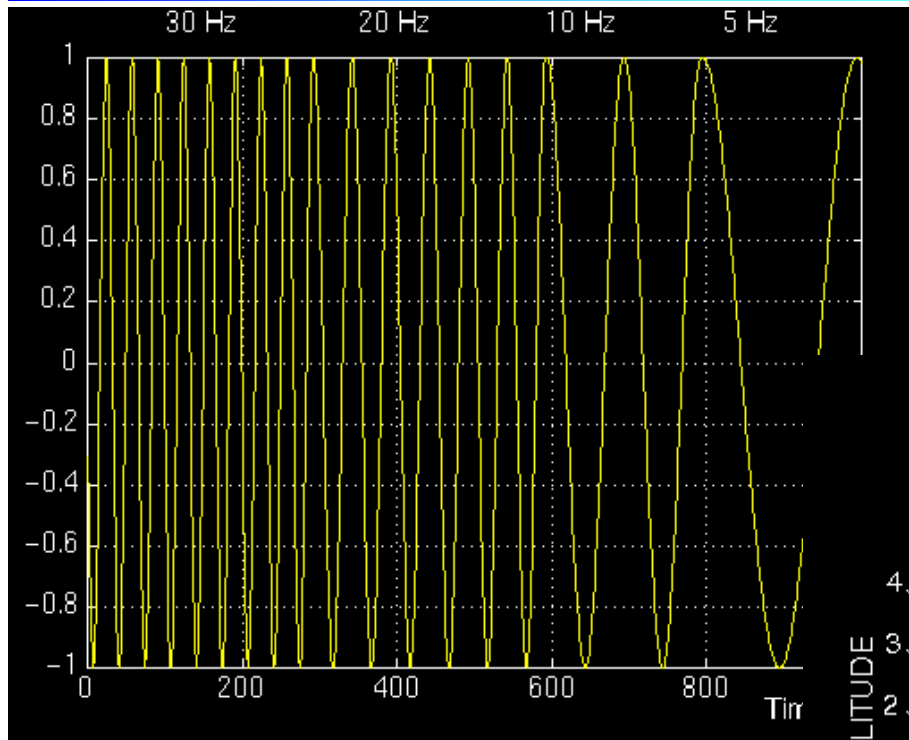
then larger-scale...



and even larger scale...



Result is indexed by translation & scale



Wavelet Transform & Basis Decomposition

We define the inner product between two functions:

$$\langle f(t), g(t) \rangle = \int_a^b f(t) \cdot g^*(t) dt$$

then the continuous wavelet transform:

$$CWT_x^\psi(\tau, s) = \Psi_x^\psi(\tau, s) = \int x(t) \cdot \psi_{\tau, s}^*(t) dt$$

can be thought of as taking the inner product between signal and all of the different wavelets (parametrized by translation & scale):

$$\psi_{\tau, s} = \frac{1}{\sqrt{s}} \psi \left(\frac{t - \tau}{s} \right)$$

Orthonormal

two functions are orthogonal iff:

$$\langle f(t), g(t) \rangle = \int_a^b f(t) \cdot g^*(t) dt = 0$$

and a set of functions is orthonormal iff:

$$\int_a^b \phi_k(t) \phi_l^*(t) dt = \delta_{kl}$$

with

$$\delta_{kl} = \begin{cases} 1 & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases}$$

Basis

If the collection of wavelets forms an **orthonormal basis**, then we can compute:

$$\mu_k = \langle f, \phi_k \rangle = \int f(t) \cdot \phi_k^*(t) dt$$

and fully reconstruct the signal from those coefficients (and knowledge of the wavelet functions) alone:

$$\begin{aligned} f(t) &= \sum_k \mu_k \phi_k(t) \\ &= \sum_k \langle f, \phi_k \rangle \phi_k(t) \end{aligned}$$

thus the transformation is reversible.

Edge Detection

Very important to both biological and computer vision:

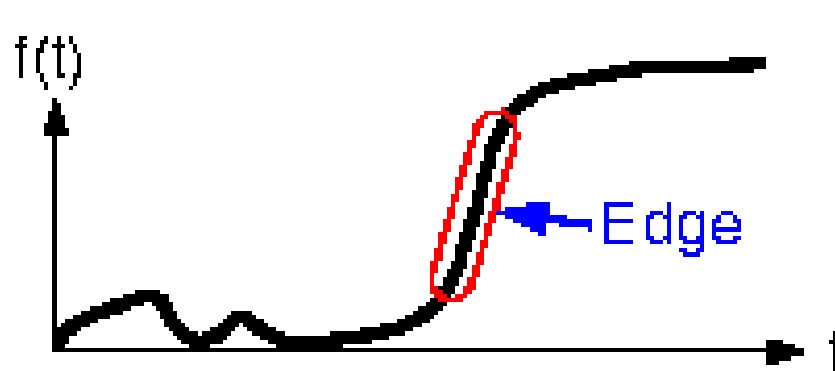
- Easy and cheap (computationally) to compute.
- Provide strong visual clues to help recognition.
- Problem: sensitive to image noise.

why? because edge detection is a high-pass filtering process and noise typically has high-pass components (e.g., speckle noise).

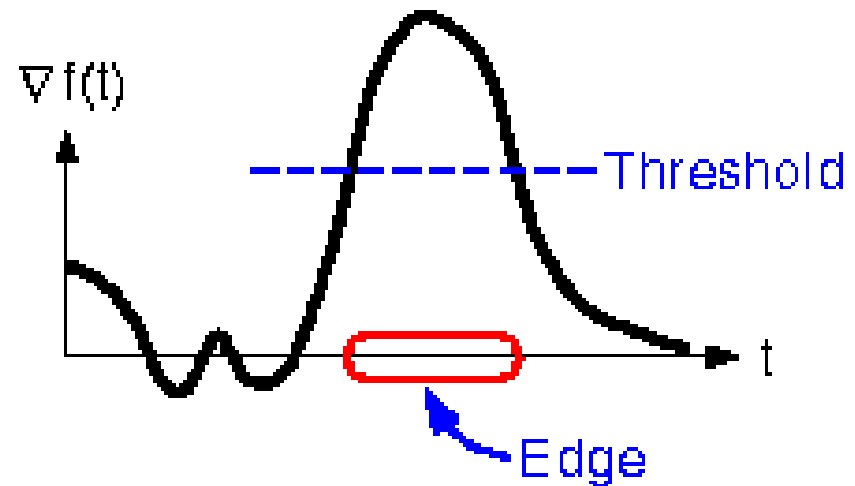
Laplacian Edge Detection

Edges are defined as zero-crossings of the second derivative (Laplacian if more than one-dimensional) of the signal.

This is very sensitive to image noise; thus typically we first blur the image to reduce noise. We then use a **Laplacian-of-Gaussian** filter to extract edges.



Smoothed signal



First derivative (gradient)

Derivatives in 2D

Gradient:

$$\begin{aligned}\frac{\partial f(x, y)}{\partial x} &= \Delta_x = \frac{f(x + d_x, y) - f(x, y)}{d_x}, \\ \frac{\partial f(x, y)}{\partial y} &= \Delta_y = \frac{f(x, y + d_y) - f(x, y)}{d_y},\end{aligned}\tag{42}$$

for discrete images:

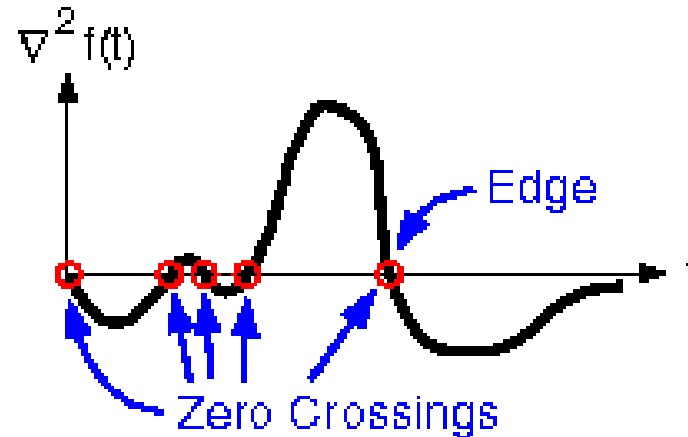
$$\begin{aligned}\Delta_x &= f(i + 1, j) - f(i, j), \\ \Delta_y &= f(i, j + 1) - f(i, j).\end{aligned}\tag{43}$$

magnitude and direction:

$$M = \sqrt{\Delta_x^2 + \Delta_y^2}, \quad \theta = \tan^{-1} \left[\frac{\Delta_y}{\Delta_x} \right].$$

Laplacian-of-Gaussian

Laplacian:

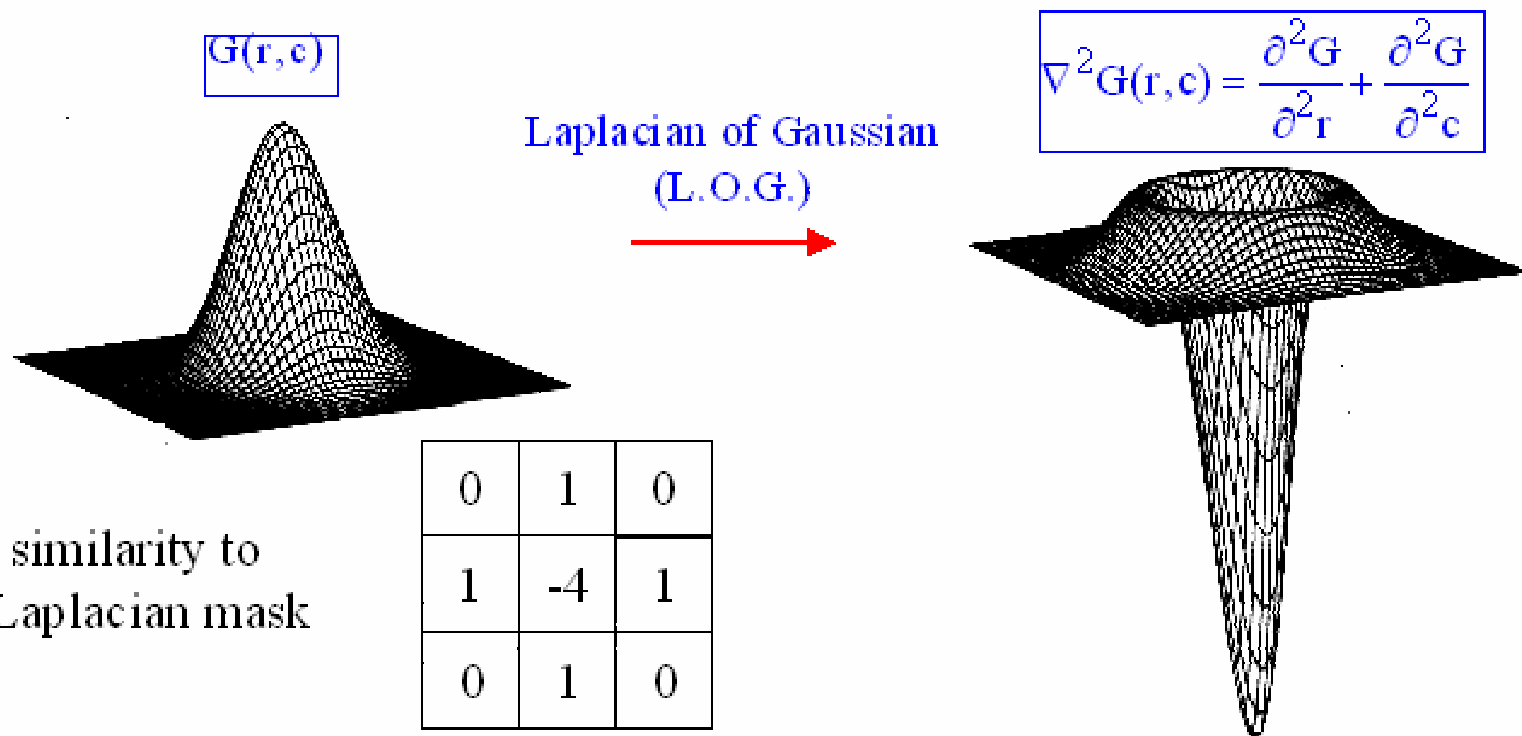


$$\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}. \quad (47)$$



Laplacian of the Gaussian

Idea: Rather than smooth image first using a Gaussian & then computing the Laplacian of the result, we can derive a mask that does both operations simultaneously



L.O.G. mask computed by sampling the L.O.G. function on a discrete grid (mask size chosen to be identical to that of the corresponding Gaussian mask)

Another Edge Detection Scheme

- Maxima of the modulus of the Gradient in the Gradient direction (Canny-Deriche):
 - use optimal 1st derivative filter to estimate edges
 - estimate noise level from RMS of 2nd derivative of filter responses
 - determine two thresholds, T_{high} and T_{low} from the noise estimate
 - edges are points which are locally maximum in gradient direction
 - a hysteresis process is employed to complete edges, i.e.,
 - edge will start when filter response $> T_{high}$
 - but may continue as long as filter response $> T_{low}$

Marr-Hildreth Edge Detection Algorithm

Given: An input image I and a value for the variance σ

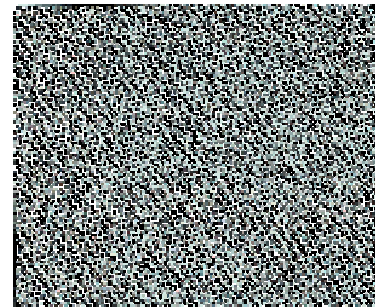
Steps:

1. Compute the L.O.G. mask corresponding to the desired value of σ
2. Apply the L.O.G. mask to image I
3. Compute zero crossings of the result
4. Label as edges all zero crossing pixels
5. Optional step: label as edges only those zero-crossings whose transition from + to - or from - to + is greater than a threshold

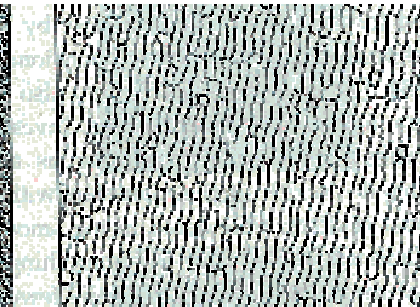
Input image



zero crossings
 $\sigma=1.0$



zero crossings
 $\sigma=5.0$

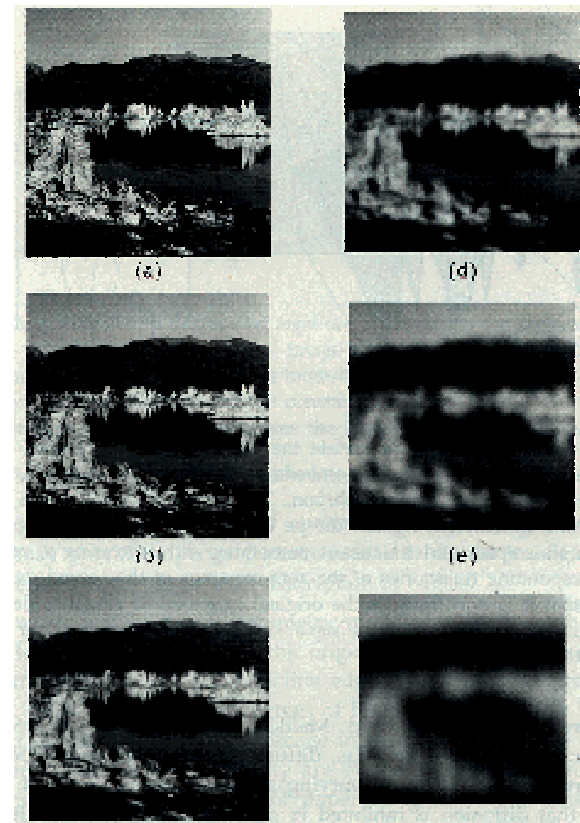


Choosing the “Right” Scale??

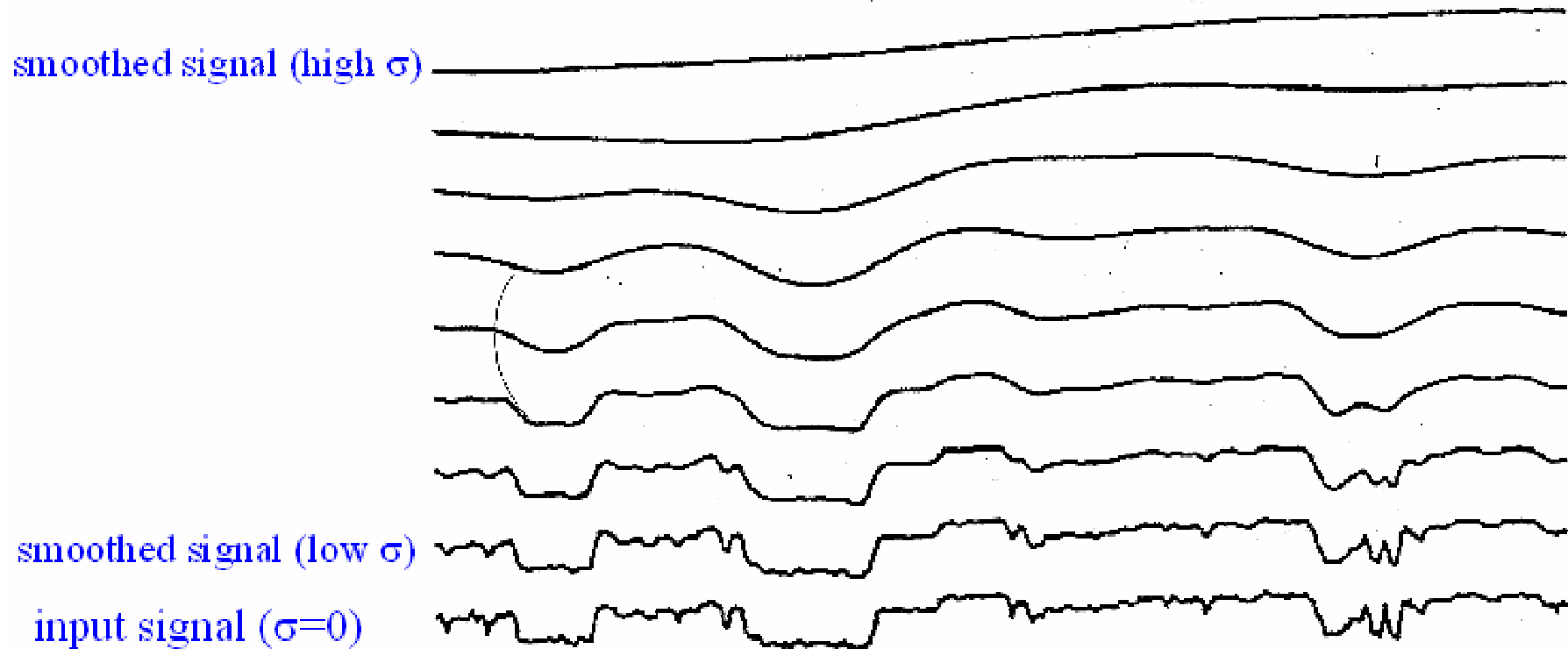
Rather than trying to decide which scale to use, we can process images at all scales & examine how the resulting image changes with increasing values of σ

Basic principles of scale-space approaches

- No single scale is “special”
- Treat s as a continuous parameter
- As s increases, we should obtain “simpler” images, i.e., no “new structures” should be created
- “Structures” that remain present for a broad range of scales signify “important” features in the image

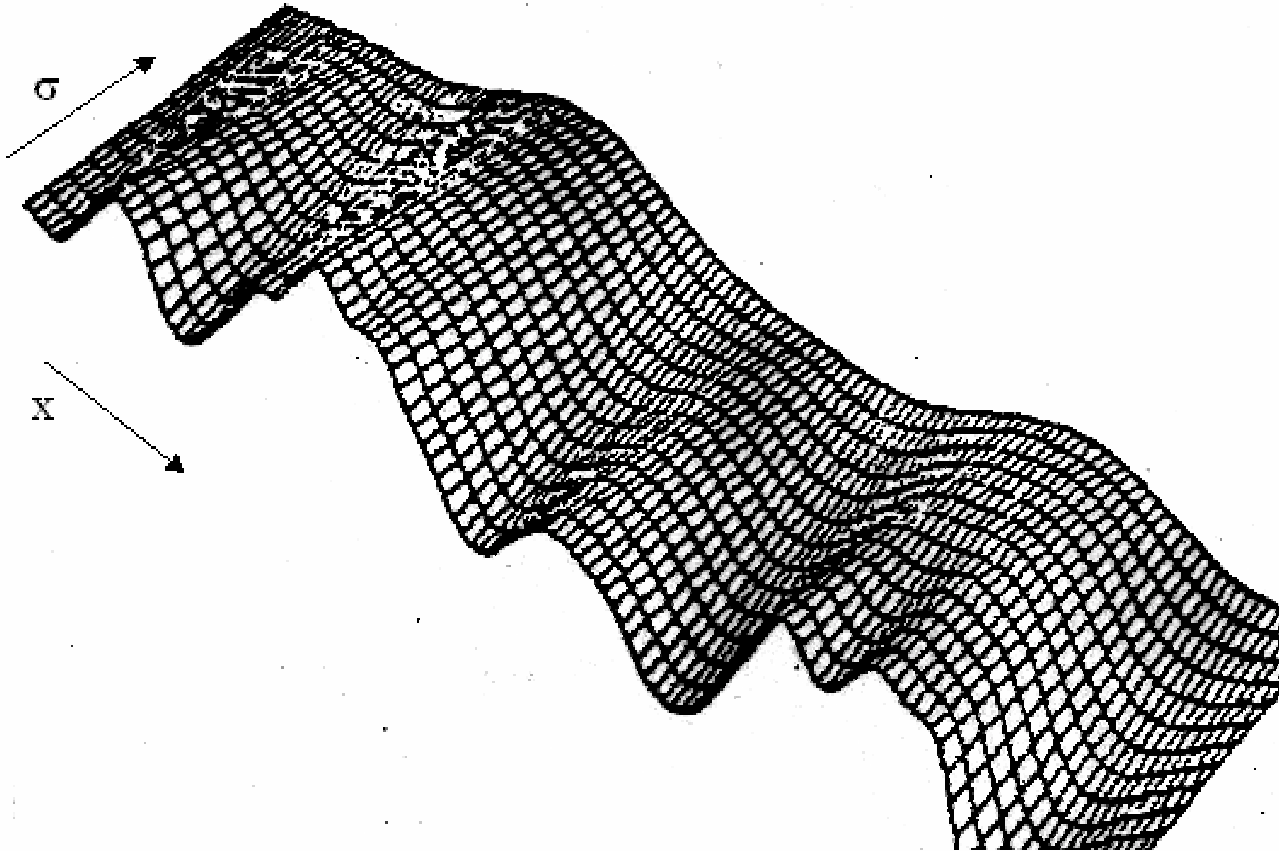


Scale Space in 1D



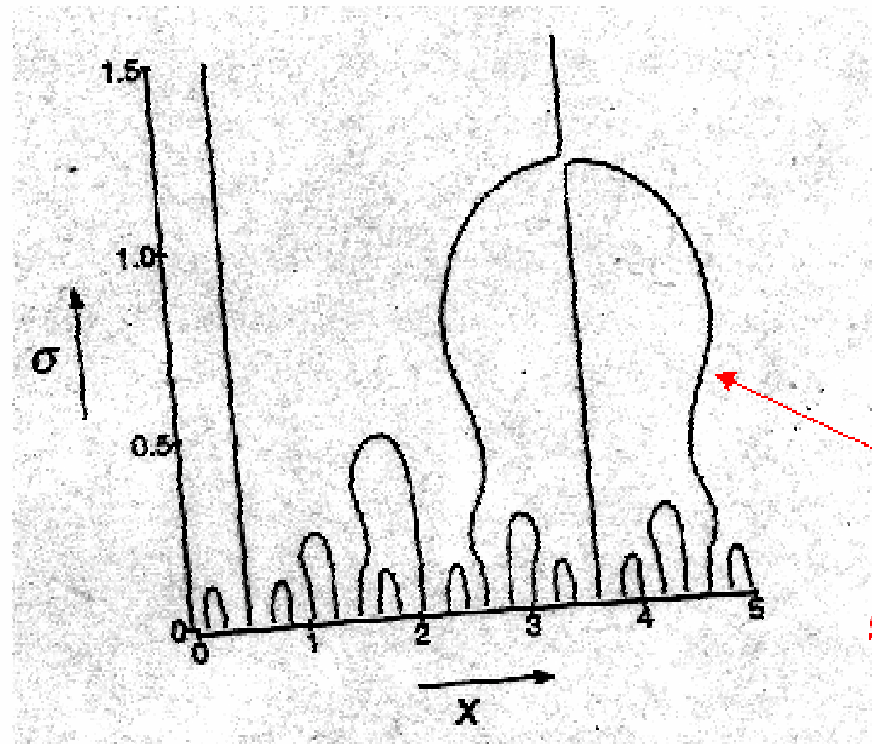
Scale Space in 1D

Resulting Scale-Space Surface



Scale Space in 1D

Idea: As the 1D image gets increasingly smoothed, the position of zero crossings moves along the smoothed curve, creating trajectory curves



Trajectory of a
single zero-crossing

- For the Gaussian function, trajectories form nested, upside-down U's
- The Gaussian smoothing function is the only function that ensures no new zero-crossings are created as σ increases
- Therefore, it is the only function for which zero crossing trajectories have the nesting property

Scale Space in 2D

- Zero crossing contours form surfaces in $(r,c)-\sigma$ space
- No new zero-crossings created as σ increases

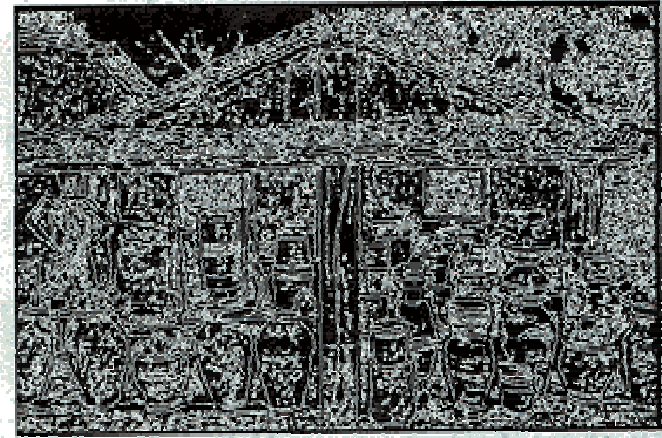
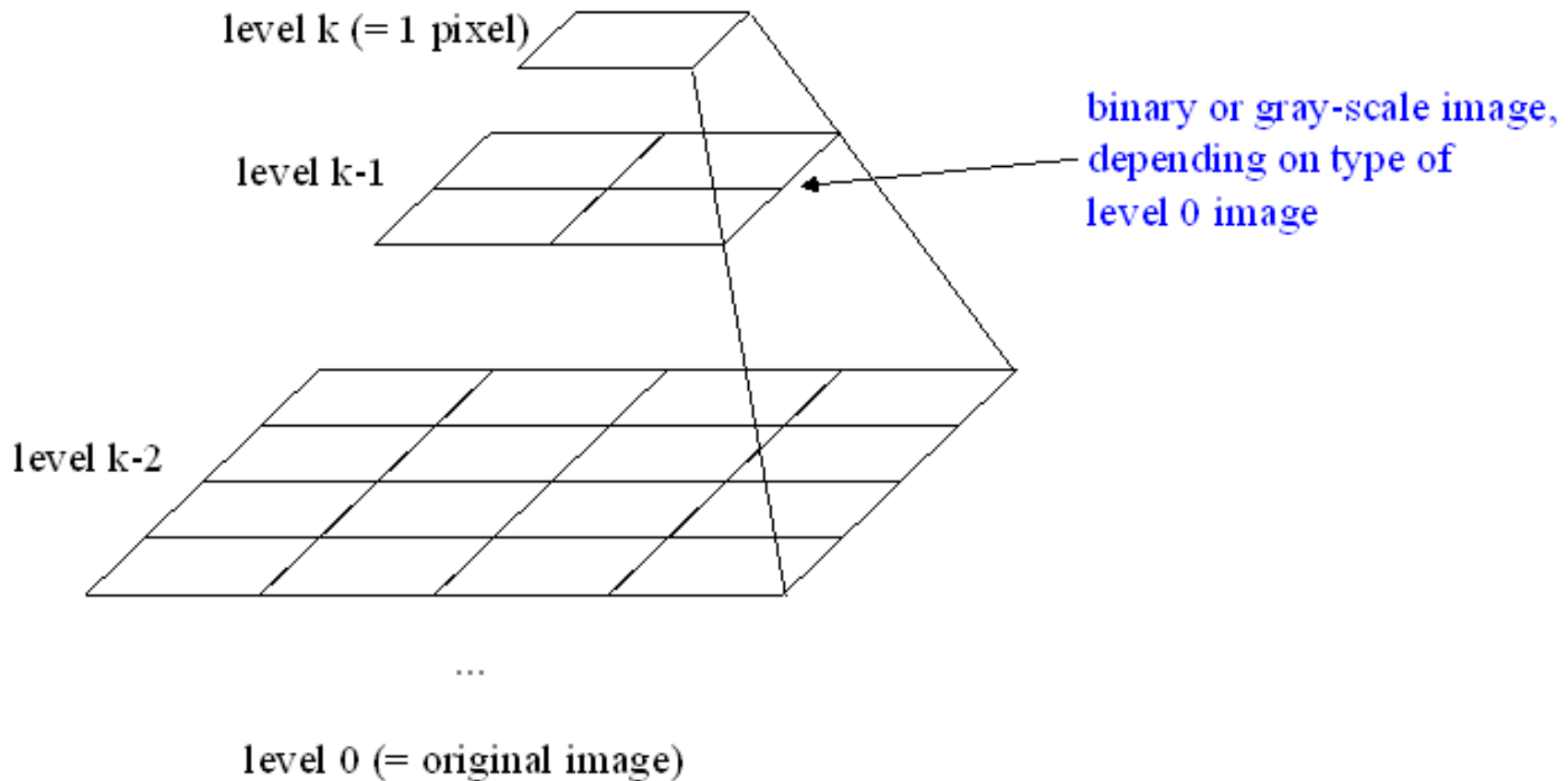


Image Pyramids

Idea: Represent $N \times N$ image as a “pyramid” of $1 \times 1, 2 \times 2, 4 \times 4, \dots, 2^k \times 2^k$ images (assuming $N = 2^k$)

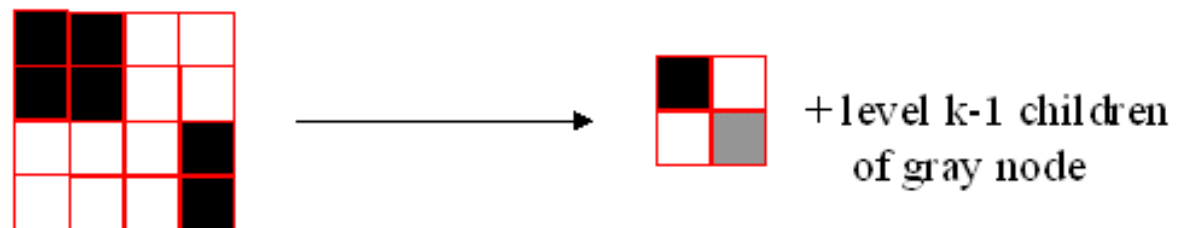


Building A Quad Tree

Two basic image operations need to be implemented:

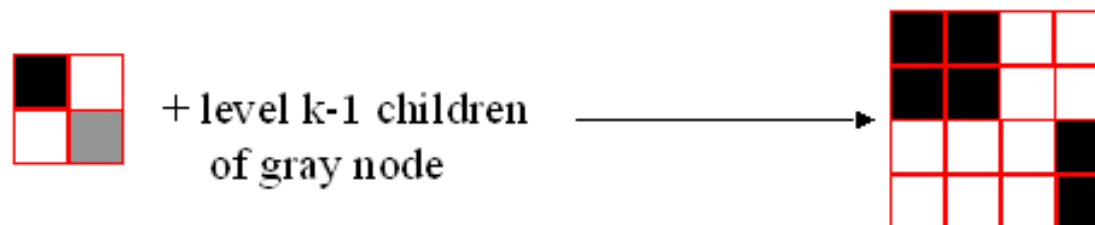
- REDUCE operation:

Map a level $k-1$ image (hi-res) to a level k image (low-res)



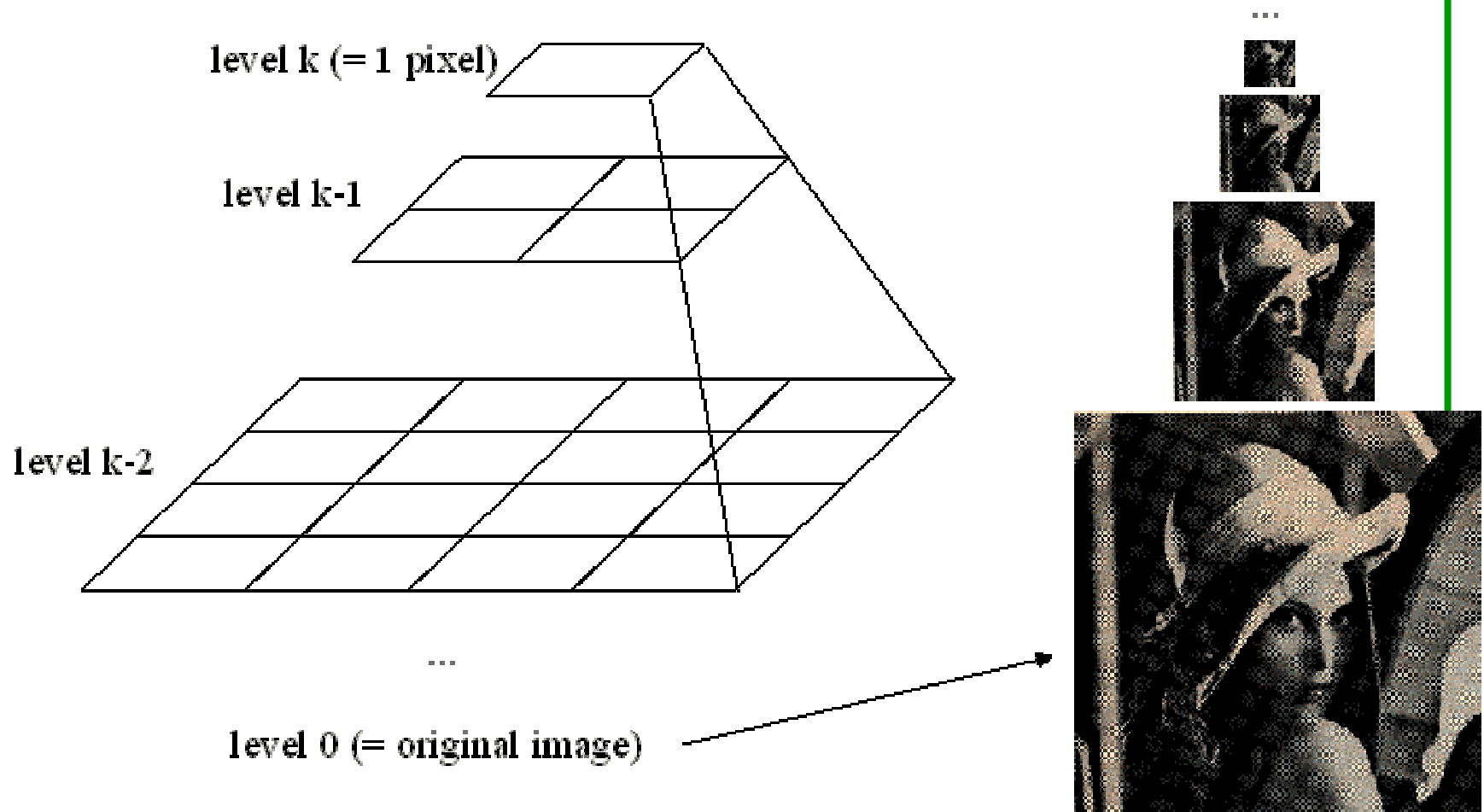
- EXPAND operation

Map a level k image (low-res) to a level $k-1$ image (hi-res)



Gaussian Pyramid

Idea: Represent $N \times N$ image as a “pyramid” of $1 \times 1, 2 \times 2, 4 \times 4, \dots, 2^k \times 2^k$ images (assuming $N = 2^k$)



Building A Gaussian Pyramid

Two basic image operations need to be specified:

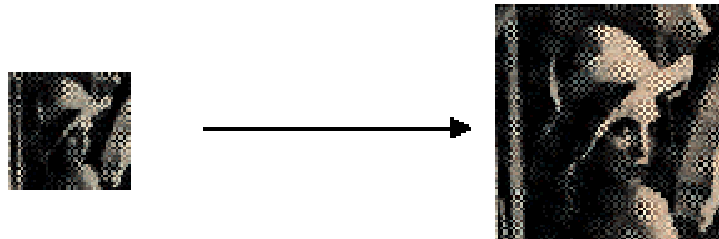
- REDUCE operation:

Map a level $k-1$ image (hi-res) to a level k image (low-res)



- EXPAND operation

Map a level k image (low-res) to a level $k-1$ image (hi-res)



Building A Gaussian Pyramid

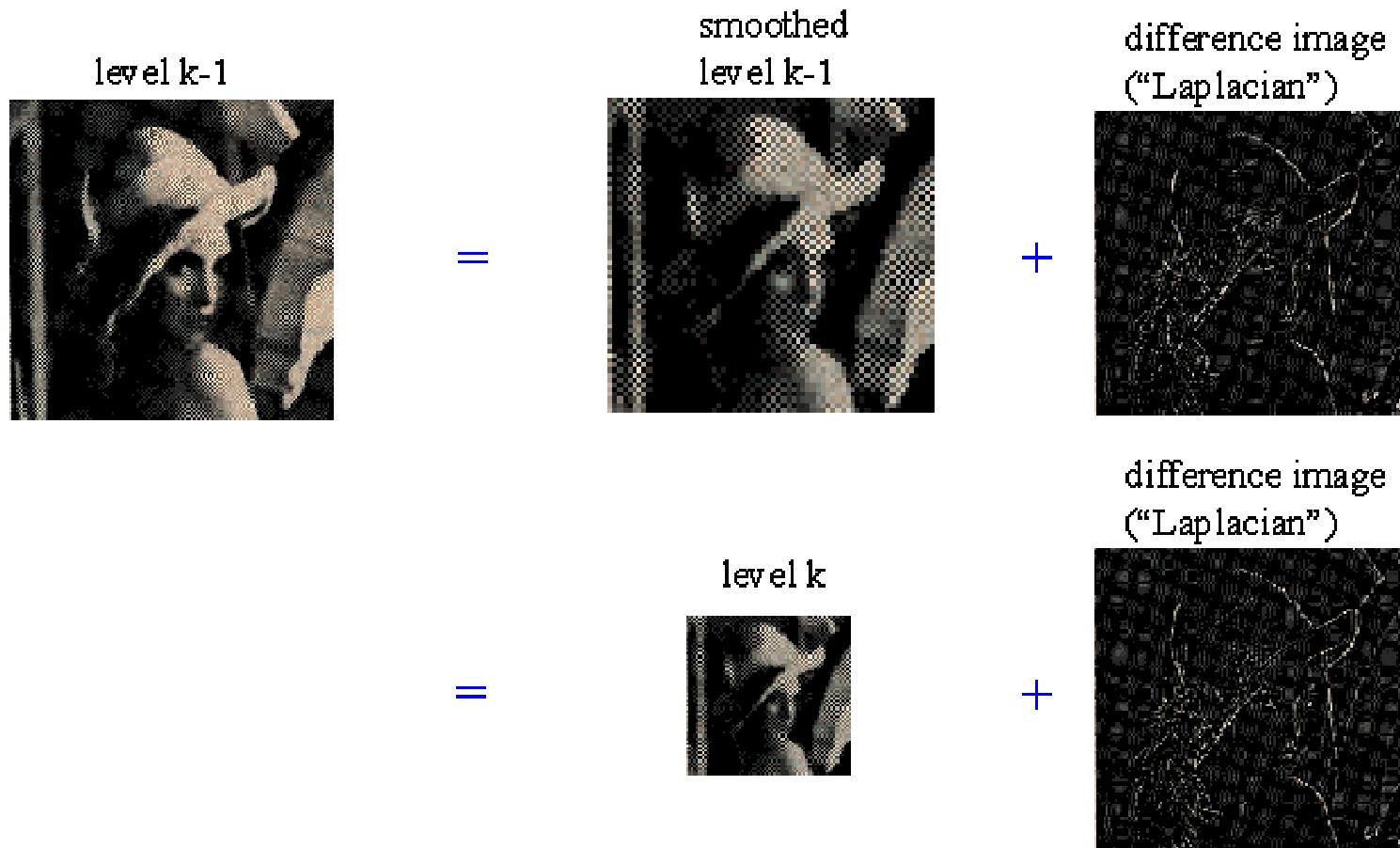
Basic idea of the REDUCE operation:

- If we smooth the level $k-1$ image appropriately, the resulting image will have less detail than the original
⇒ we can represent the result with a lower-resolution image!!



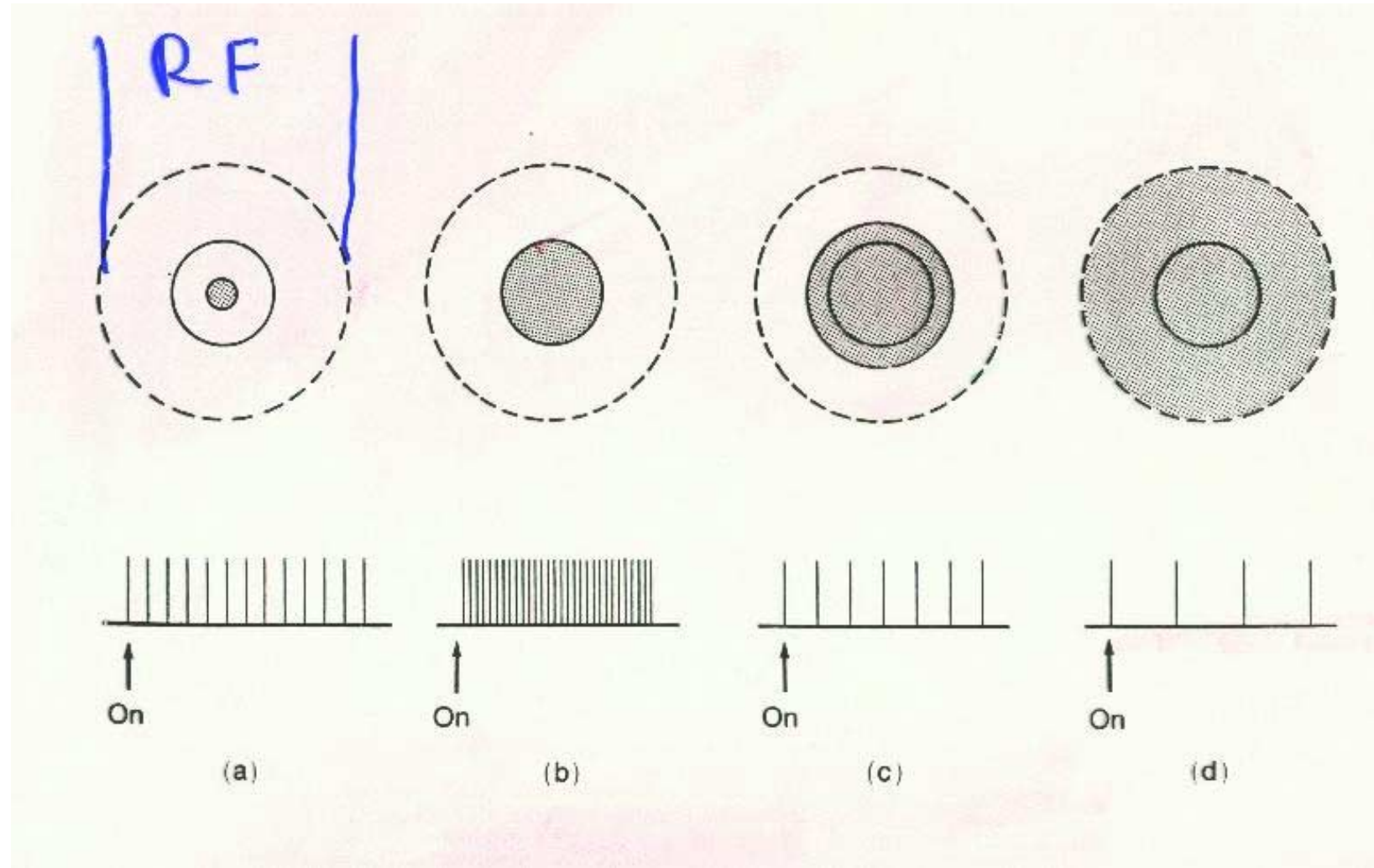
Laplacian Image Pyramid

Idea: Representing level k-1 image by a level k image & a "difference image"



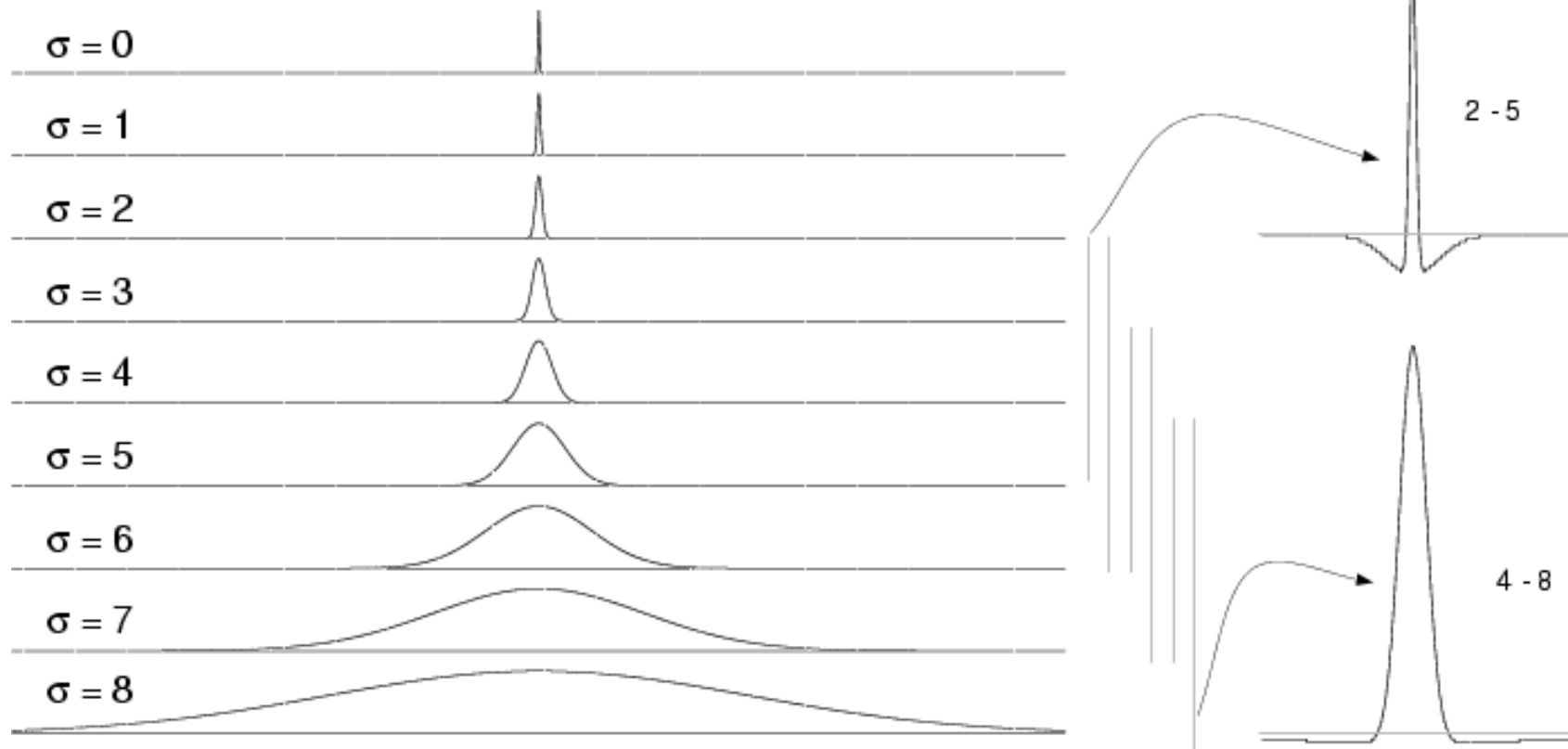
Biological Feature Extraction

Center-surround vs. Laplacian of Gaussian.



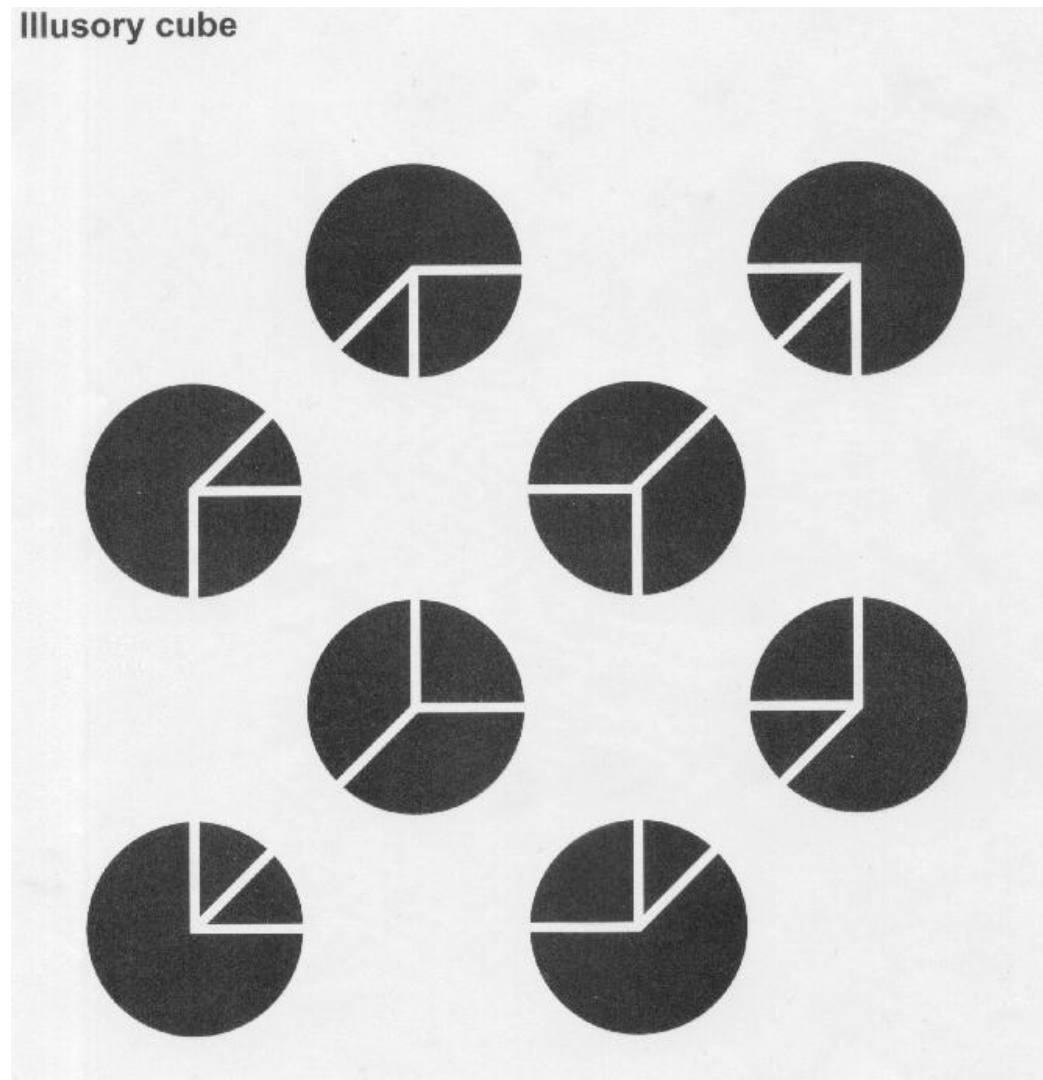
Using Pyramids to Compute Biological Features

- Build Gaussian Pyramid
- Take difference between pixels at same image locations but different scales
- Result: difference-of-Gaussians receptive fields



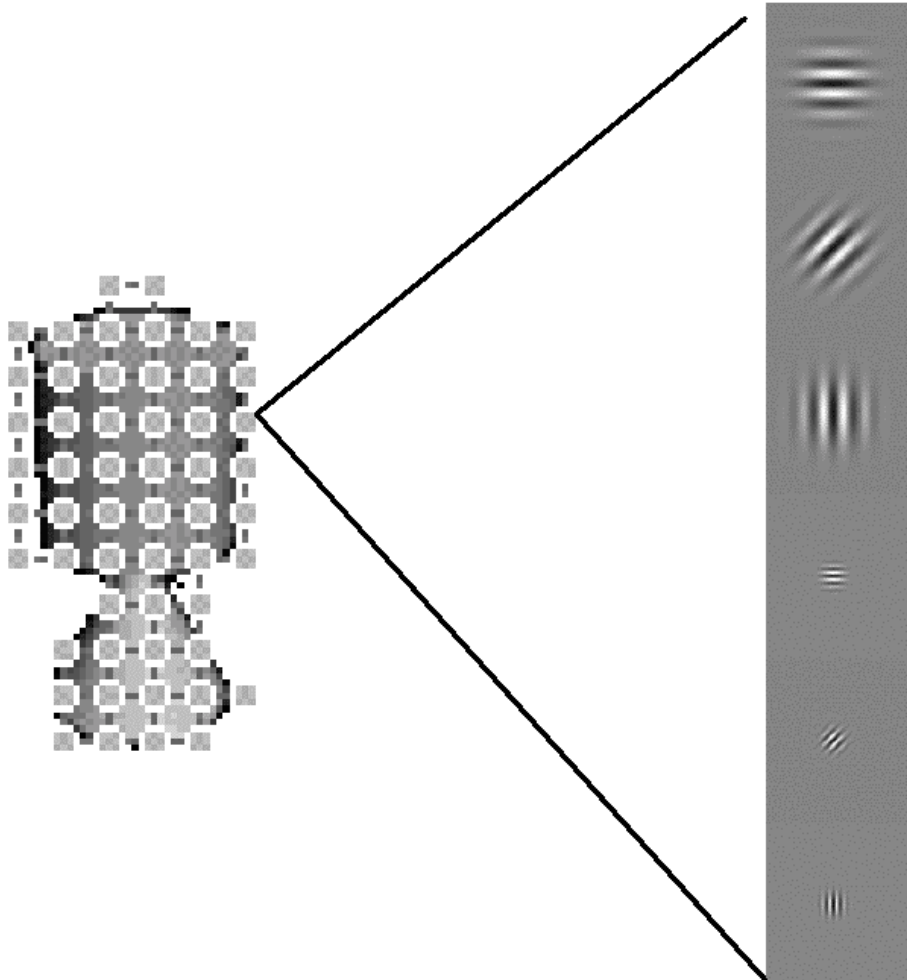
Illusory Contours

Some mechanism is responsible for our illusory perception of contours where there are none...



Gabor “jets”

Similar to a biological hypercolumn: collection of Gabor filters with various orientations and scales, but all centered at one visual location.



Non-Classical Surround

Sillito et al, Nature, 1995: response of neurons is modulated by stimuli outside the neuron's receptive field.

Method:

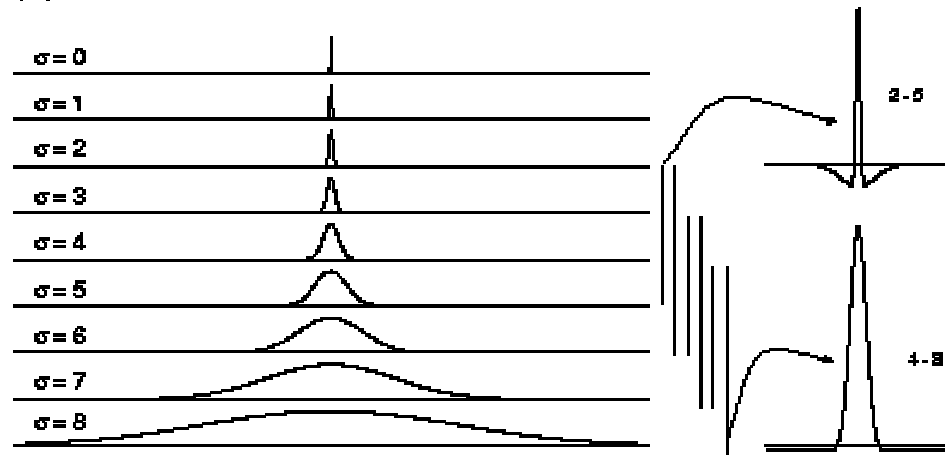
- Map receptive field location and size
- Check that neuron does not respond to stimuli outside the mapped RF
- Present stimulus in RF
- Compare this baseline response to response obtained when stimuli are also present outside the RF.

Result:

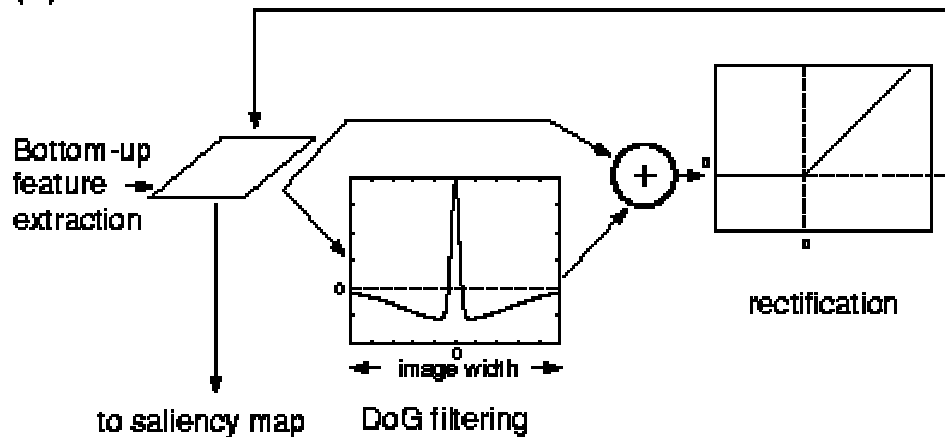
- stimuli outside RF similar to the one inside RF inhibit neuron
- stimuli outside RF very similar to the one inside RF do not affect (or enhance very slightly) neuron

Non-classical surround inhibition

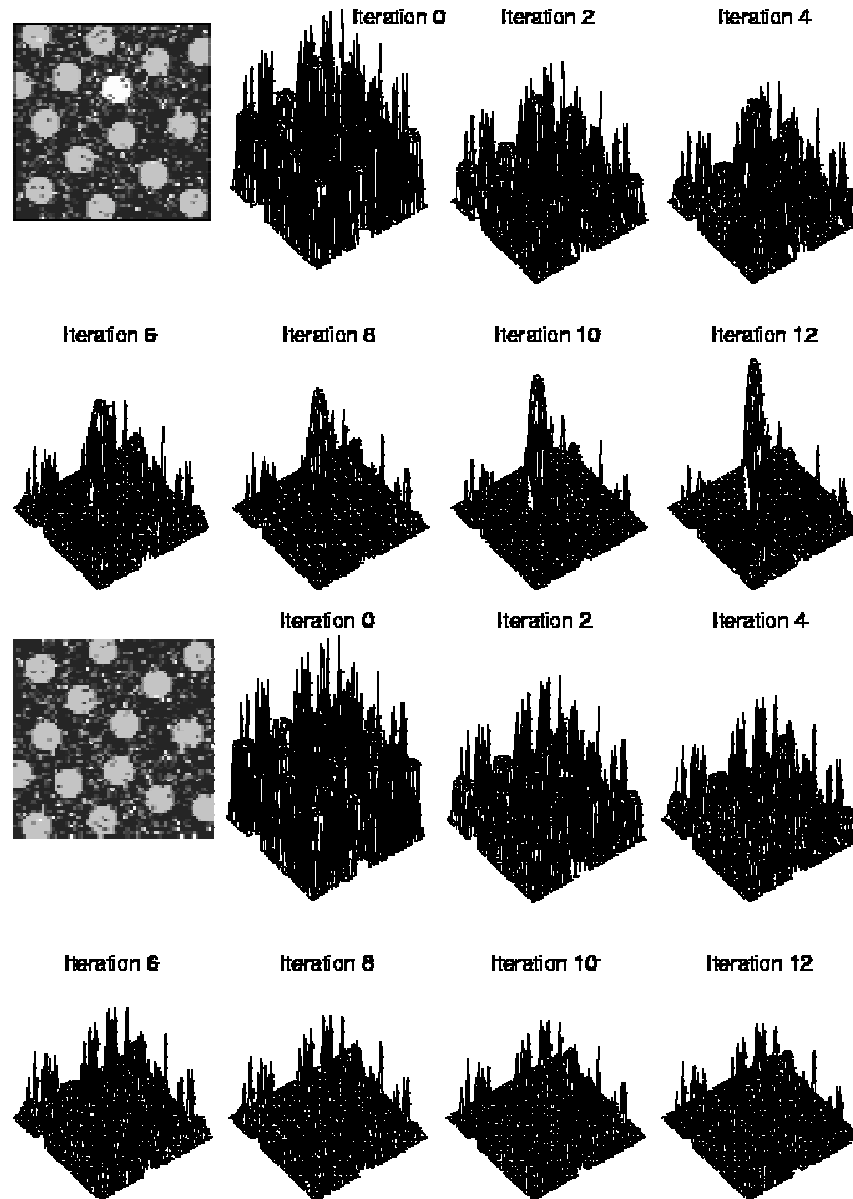
(a)



(b)



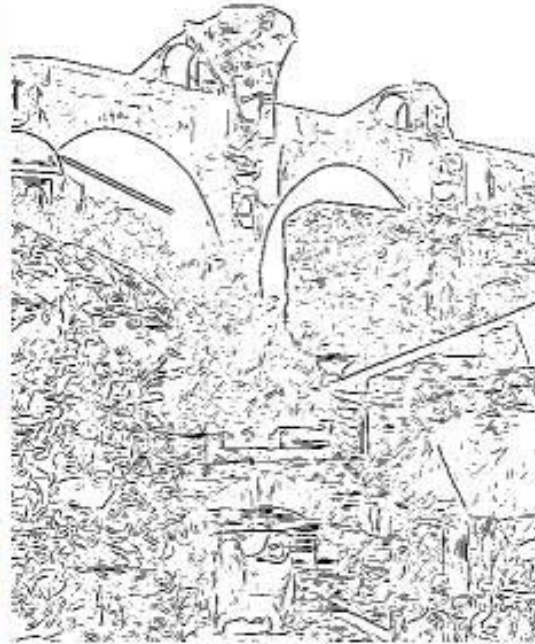
Example



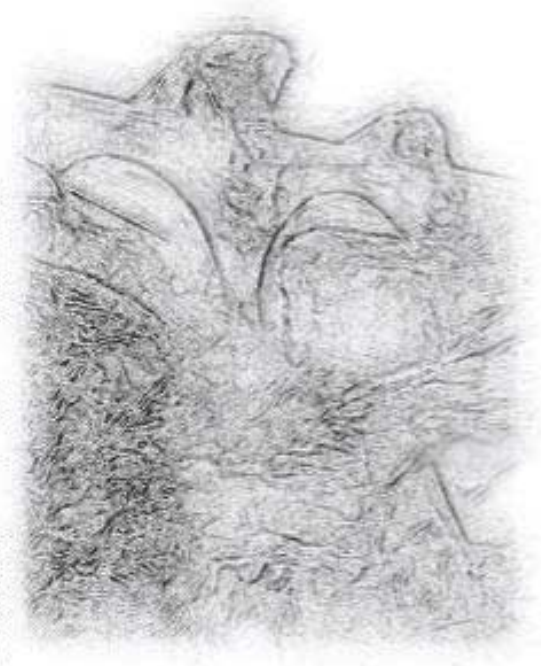
Non-Classical Surround & Edge Detection



(a) Original image



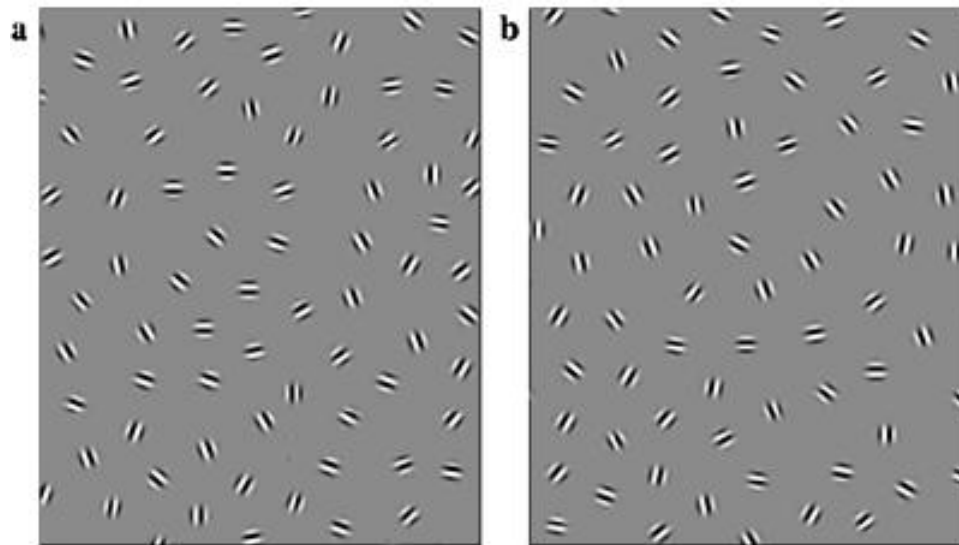
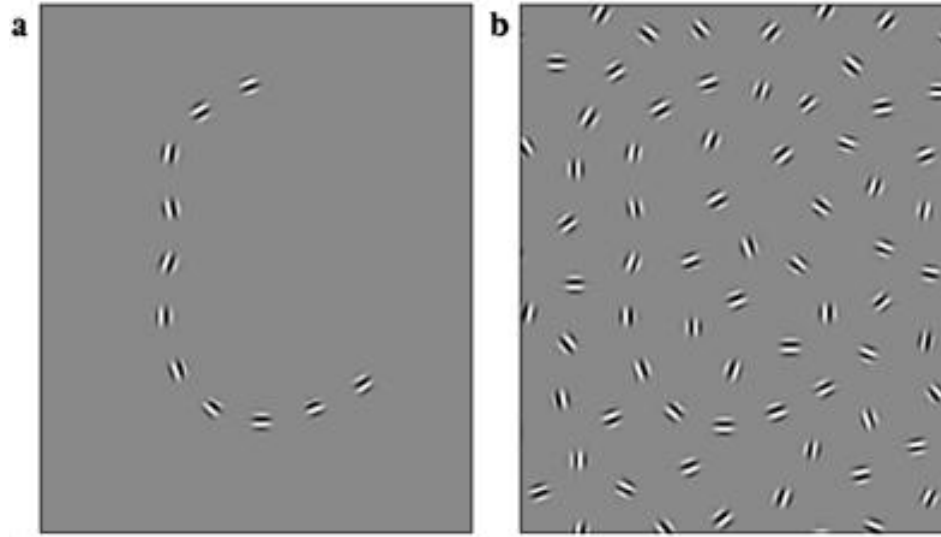
(b) Classical (local input)



(c) Non-classical (nonlocal) input only

Holt & Mel, 2000

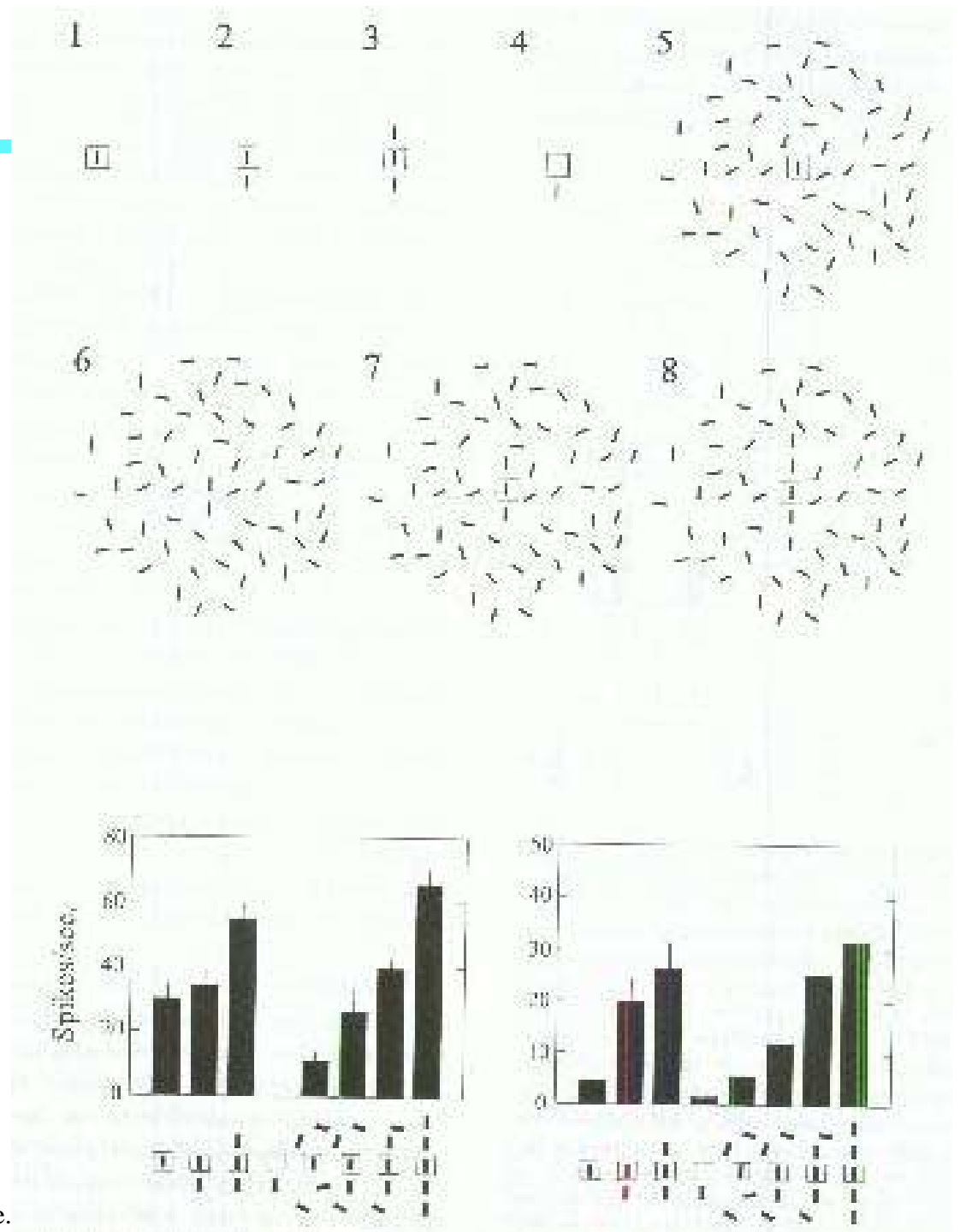
Long-range Excitation



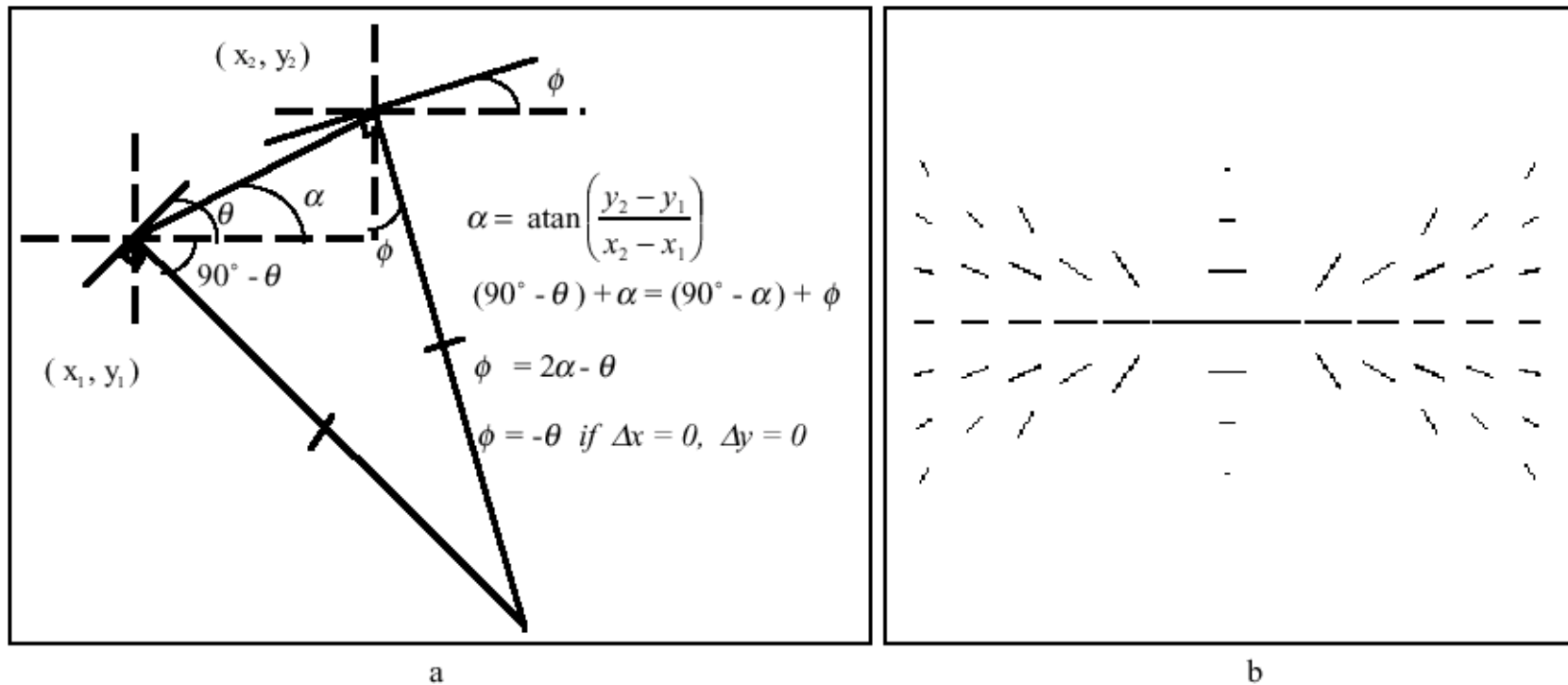
Long-range

Gilbert et al, 2000.

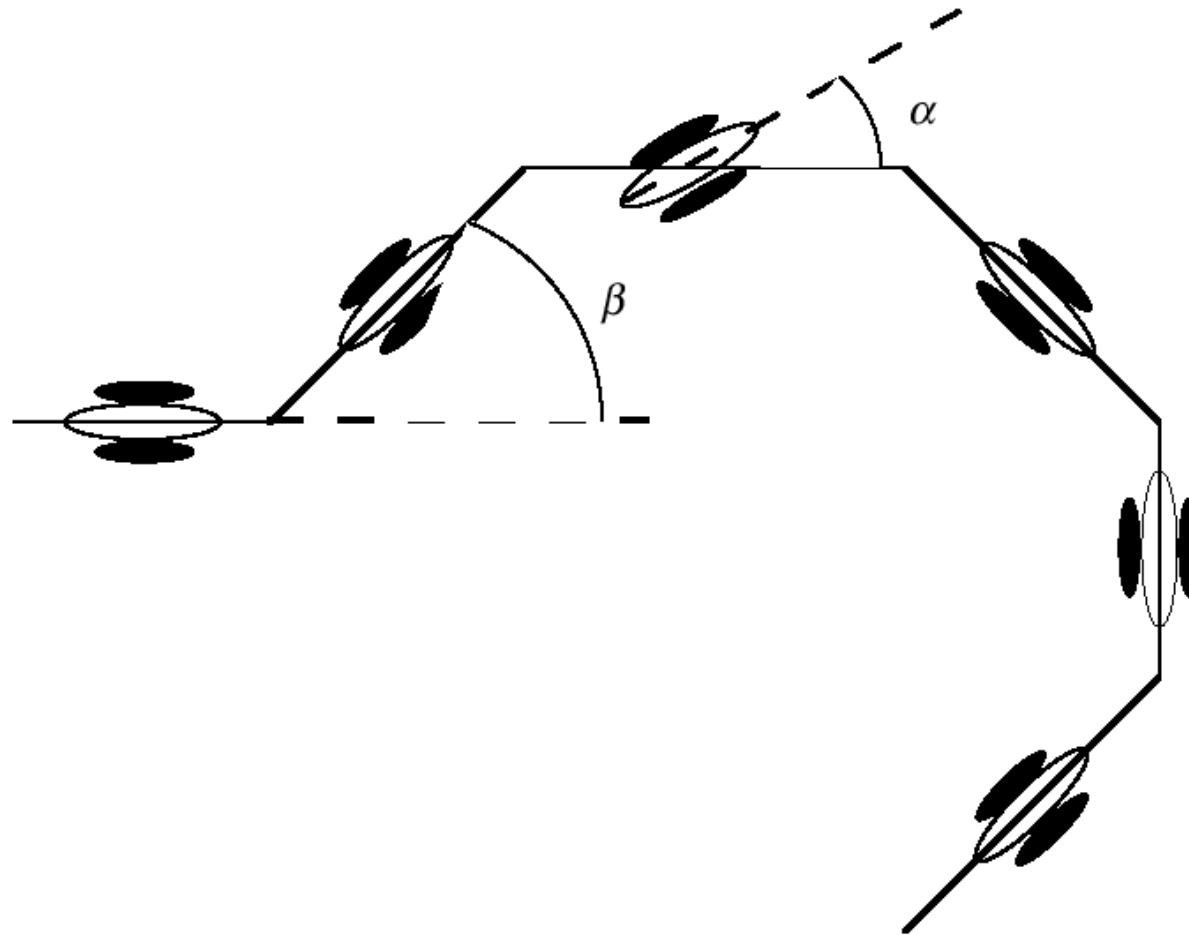
Stimulus outside RF enhances neuron's response if placed and oriented such as to form a contour.



Modeling long-range connections



Contour completion



Grouping and Object Segmentation

We can do much more than simply extract and follow contours...

