**Solution:**

**Question1**

1. **"Every graduate student is committed to academic honesty."**

   Ay (Graduate(y) -> CommitedTo(y, AcademicHonesty)), or

   Ex AcademicHonesty(x) & Ay (Graduate(y) -> CommitedTo(y, x))

   *Or assuming that small alphabet stands for variable:*

   Graduate(y) -> CommitedTo(y, AcademicHonesty)

   Ex AcademicHonesty(x) & (Graduate(y) -> CommitedTo(y, x))

2. **"Only Bill can get his car started."**

   There is two different readings leading to two different answers:

   <u>First reading: "Bill is the only individual X such that X can get Bill's car started."</u>

   CanStart(Bill, car(Bill)) & Ax (CanStart(x, car(Bill)) -> x = Bill), or

   Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x) & Ay (CanStart(y, x) -> x = Bill))

   CanStart(Bill, car(Bill)) & (CanStart(x, car(Bill)) -> x = Bill), or

   Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x) & (CanStart(y, x) -> x = Bill))

   The below sentence is also complete:

   Ax (CanStart(x, car(Bill)) -> x = Bill)

   CanStart(x, car(Bill)) -> x = Bill

   <u>Second reading: "Bill is the only individual X such that X can get X's car started."</u>

   CanStart(Bill, car(Bill)) & Ax (CanStart(x, car(x)) -> x = Bill), or

   Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x)) & Ax ((Ey Car(y) & Owns(x, y) & CanStart(x, y)) -> x = Bill)

   CanStart(Bill, car(Bill)) & (CanStart(x, car(x)) -> x = Bill), or

Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x)) & ((Ey Car(y) & Owns(x, y) & CanStart(x, y)) -> x = Bill)

Ax (CanStart(x, car(x)) -> x = Bill)

CanStart(x, car(x)) -> x = Bill

3. **"Things near the earth fall to the ground unless something holds them up."**

   Ez Ax (Near(x, Earth) & ~HoldsUp(z, x)=>FallsTo(x,Ground)) or

   They can use Ey Earth(y) and Et Ground(t) and continue the same stuff.

   Ax (Near(x, Earth) -> (FallsTo(x, Ground) | Ez HoldsUp(z, x)))

4. **"Dinner is available only if booked in advance for at least two persons."**

   Ad Ap Party(p) & Dinner(d) & BookedFor(d, p) & sizeOf(p)>=2 => AvailableFor(d, p)

5. **"No man helps another without helping himself."**

   It means that if you help someone else, then you have helped yourself.

   Ax Ey (Human(x) & Helps(x, y)) -> Helps(x, x)

   Ax Ey Helps(x, y) -> Helps(x, x)

   ~ (ExEy (Human(x) & Helps(x, y)) -> ~Helps(x, x))

   I hope everyone understand the same thing from this sentence.

## Question 2

1) Ex Ay ~Brother(x,y)
   ~VxEy Brother(x,y)

2) Ax,y Sister(x,y)=>Female(y)

3) Ax,y,f,m (Mother(x,m) && Father(x,f) && Mother(y,m) && Father(y,f)) =>
   (Sister(x,y) || Brother(x,y))

4) Ax,y, (Cousin(x,y) <=>
   (Ef1,m1,f2,m2 Father(x,f1) && Mother(x,m1) && Father(y,f2) &&
   Father(y,m2) &&

(Brother(f1,f2) || Brother(m1,f2) || Sister(m1,f2) || Sister(m1,m2))

**Question 3**

**For this question there might be different orderings in using the sentences:**

Step 1 convert to CNF

1. $\neg ice\_cream(x) \vee food(x)$

2. $\neg fudge(x) \vee food(x)$

3. $\neg food(x) \vee \neg food(y) \vee \neg cold(x) \vee \neg combine(x, y) \vee cold(y)$

Note that we can convert the statement

$$\exists x\ \exists y\ \ ice\_cream(x) \wedge cold(x) \wedge fudge(y) \wedge combine(x, y)$$

as follows: Start by replacing existential variables with Skolem constants $MysteryIceCream$ for $x$, and $MysteryFudge$ for $y$.

$ice\_cream(MysteryIceCream) \wedge cold(MysteryIceCream) \wedge fudge(MysteryFudge) \wedge combine(MysteryIceCream, MysteryFudge)$

Use AND elimination to break the above statement into four new ones:

4. $ice\_cream(MysteryIceCream)$

5. $cold(MysteryIceCream)$

6. $fudge(MysteryFudge)$

7. $combine(MysteryIceCream, MysteryFudge)$

Negated Query: $\neg \exists x\ \ (fudge(x) \wedge cold(x))$

8. $\neg fudge(x) \vee \neg cold(x)$

Step 2 Resolution with refutation

Combine 8 and 6 with resolution using substitution $x/MysteryFudge$

9. $\neg cold(MysteryFudge)$

Combine 1 and 4 with resolution using substitution $x/MysteryIceCream$

10. $food(MysteryIceCream)$

Combine 2 and 6 with resolution using substitution $x/MysteryFudge$

11. $food(MysteryFudge)$

Combine 11 and 3 with resolution using substitution $y/MysteryFudge$

12. $\neg food(x) \vee \neg cold(x) \vee \neg combine(x, MysteryFudge) \vee cold(MysteryFudge)$

Combine 12 and 10 with resolution using substitution $x/MysteryIceCream$

13. $\neg cold(MysteryIceCream) \vee \neg combine(MysteryIceCream, MysteryFudge) \vee cold(MysteryFudge)$

Combine 13 and 5 with resolution

14. $\neg combine(MysteryIceCream, MysteryFudge) \vee cold(MysteryFudge)$

Combine 14 and 7 with resolution

15. $cold(MysteryFudge)$

Combine 15 and 9 with resolution which results in a contradiction.

Therefore there exists cold fudge.

**Question 4**

**a)** $p \longrightarrow q \Leftrightarrow \sim p \vee q \Leftrightarrow \sim (p \wedge \sim q)$

**b)** $p \wedge q \Leftrightarrow \sim (\sim p \vee \sim q)$

**c)** $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \Leftrightarrow \sim ((p \rightarrow q) \rightarrow \sim (q \rightarrow p))$