Logical reasoning systems

- Theorem provers and logic programming languages
- Production systems
- Frame systems and semantic networks
- Description logic systems

Logical reasoning systems

- Theorem provers and logic programming languages – Provers: use resolution to prove sentences in full FOL. Languages: use backward chaining on restricted set of FOL constructs.
- Production systems – based on implications, with consequents interpreted as action (e.g., insertion & deletion in KB). Based on forward chaining + conflict resolution if several possible actions.
- Frame systems and semantic networks – objects as nodes in a graph, nodes organized as taxonomy, links represent binary relations.
- Description logic systems – evolved from semantic nets. Reason with object classes & relations among them.

Basic tasks

- Add a new fact to KB – TELL
- Given KB and new fact, derive facts implied by conjunction of KB and new fact. In forward chaining: part of TELL
- Decide if query entailed by KB – ASK
- Decide if query explicitly stored in KB – restricted ASK
- Remove sentence from KB: distinguish between correcting false sentence, forgetting useless sentence, or updating KB re. change in the world.

Indexing, retrieval & unification

- Implementing sentences & terms: define syntax and map sentences onto machine representation.

  Compound: has operator & arguments.

  e.g., $c = P(x) \wedge Q(x)$ $Op[c] = \wedge$; $Args[c] = [P(x), Q(x)]$

- FETCH: find sentences in KB that have same structure as query.

  ASK makes multiple calls to FETCH.

- STORE: add each conjunct of sentence to KB. Used by TELL.

  e.g., implement KB as list of conjuncts

  TELL(KB, $A \wedge \neg B$) TELL(KB, $\neg C \wedge D$)

  then KB contains: [A, $\neg$B, $\neg$C, D]

Complexity

- With previous approach,

  FETCH takes O(n) time on n-element KB

  STORE takes O(n) time on n-element KB (if check for duplicates)

Faster solution?

Table-based indexing

- **What are you indexing on? Predicates (relations/functions). Example:**

Key	Positive	Negative	Conclu-sion	Premise
Mother	Mother(ann,sam) Mother(grace,joe)	-Mother(ann,al)	xxxx	xxxx
dog	dog(rover) dog(fido)	-dog(alice)	xxxx	xxxx

Table-based indexing

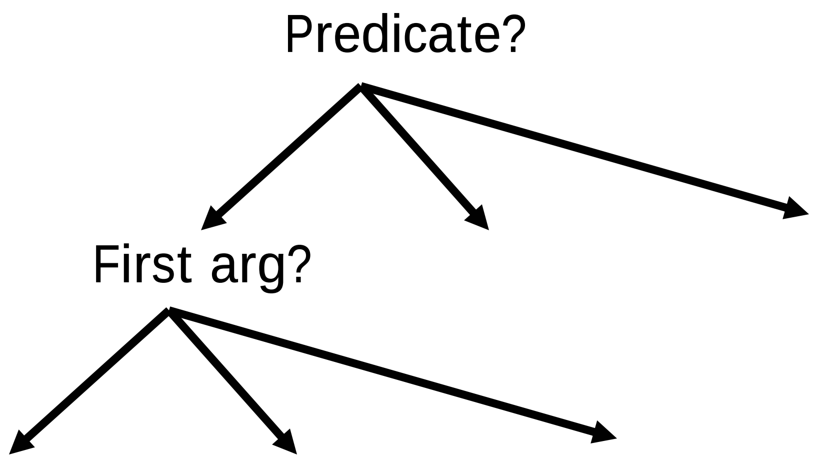
- Use hash table to avoid looping over entire KB for each TELL or FETCH

  e.g., if only allowed literals are single letters, use a 26-element array to store their values.

- More generally:
  - convert to Horn form
  - index table by predicate symbol
  - for each symbol, store:
    - list of positive literals
    - list of negative literals
    - list of sentences in which predicate is in conclusion
    - list of sentences in which predicate is in premise

Tree-based indexing

- Hash table impractical if many clauses for a given predicate symbol
- Tree-based indexing (or more generally combined indexing): compute indexing key from predicate and argument symbols



Tree-based indexing

Example:

Person(age,height,weight,income)

Person(30,72,210,45000)

Fetch(Person(age,72,210,income))

Fetch(Person(age,height>72,weight<210,income))

Unification algorithm: Example

Understands(mary,x) implies Loves(mary,x)

Understands(mary,pete) allows the system to substitute pete for x and make the implication that IF

Understands(mary,pete) THEN Loves(mary,pete)

Unification algorithm

- Using clever indexing, can reduce number of calls to unification

- Still, unification called very often (at basis of modus ponens) => need efficient implementation.

- See AIMA p. 303 for example of algorithm with O(n^2) complexity

  (n being size of expressions being unified).

Logic programming

Remember: knowledge engineering vs. programming...

Sound bite: computation as inference on logical KBs

	Logic programming	Ordinary programming
1.	Identify problem	Identify problem
2.	Assemble information	Assemble information
3.	Tea break	Figure out solution
4.	Encode information in KB	Program solution
5.	Encode problem instance as facts	Encode problem instance as data
6.	Ask queries	Apply program to data
7.	Find false facts	Debug procedural errors

Should be easier to debug $Capital(NewYork, US)$ than $x := x + 2$!

Logic programming systems

e.g., **Prolog:**

- Program = sequence of sentences (implicitly conjoined)
- All variables implicitly universally quantified
- Variables in different sentences considered distinct
- Horn clause sentences only (= atomic sentences or sentences with no negated antecedent and atomic consequent)
- Terms = constant symbols, variables or functional terms
- Queries = conjunctions, disjunctions, variables, functional terms
- Instead of negated antecedents, use negation as failure operator: goal NOT P considered proved if system fails to prove P
- Syntactically distinct objects refer to distinct objects
- Many built-in predicates (arithmetic, I/O, etc)

Prolog systems

Basis: backward chaining with Horn clauses + bells & whistles
Widely used in Europe, Japan (basis of 5th Generation project)
Compilation techniques $\Rightarrow$ 10 million LIPS

Program = set of clauses = `head :- literal`$_1$`, ... literal`$_n$`.`
Efficient unification by open coding
Efficient retrieval of matching clauses by direct linking
Depth-first, left-to-right backward chaining
Built-in predicates for arithmetic etc., e.g., `X is Y*Z+3`
Closed-world assumption ("negation as failure")
  e.g., `not PhD(X)` succeeds if `PhD(X)` fails

Basic syntax of facts, rules and queries

```
<fact>  ::= <term> .
<rule>  ::= <term> :- <term> .
<query> ::= <term> .
<term>  ::= <number> | <atom> | <variable>
          | <atom> (<terms>)
<terms> ::= <term> | <term>, <terms>
```

A PROLOG Program

- A PROLOG program is a set of ***facts*** and ***rules***.
- A simple program with just facts :

```
parent(alice, jim).
parent(jim,   tim).
parent(jim,   dave).
parent(jim,   sharon).
parent(tim,   james).
parent(tim,   thomas).
```

A PROLOG Program

- c.f. a table in a relational database.
- Each line is a ***fact*** (a.k.a. a tuple or a row).
- Each line states that some person X is a parent of some (other) person Y.
- In GNU PROLOG the program is kept in an ASCII file.

A PROLOG Query

- Now we can ask PROLOG questions :

```
| ?- parent(alice, jim).
yes
| ?- parent(jim, herbert).
no
| ?-
```

A PROLOG Query

- Not very exciting. But what about this :

```
| ?- parent(alice, Who).
Who = jim
yes
| ?-
```

- `Who` is called a ***logical variable***.
  - PROLOG will set a logical variable to any value which makes the query succeed.

A PROLOG Query II

- Sometimes there is more than one correct answer to a query.
- PROLOG gives the answers one at a time. To get the next answer type `;`.

```
| ?- parent(jim, Who).
Who = tim ? ;
Who = dave ? ;
Who = sharon ? ;
yes
| ?-
```

NB : The `;` do not actually appear on the screen.

A PROLOG Query II

```
| ?- parent(jim, Who).
Who = tim ? ;
Who = dave ? ;
Who = sharon ? ;
yes
| ?-
```

NB : The ; do not actually appear on the screen.

- After finding that `jim` was a parent of `sharon` GNU PROLOG detects that there are no more alternatives for `parent` and ends the search.

conjunction

Prolog example

Depth-first search from a start state X:

```
dfs(X) :- goal(X).
dfs(X) :- successor(X,S),dfs(S).
```

No need to loop over S: successor succeeds for each

Appending two lists to produce a third:

```
append([],Y,Y).
append([X|L],Y,[X|Z]) :- append(L,Y,Z).
```

```
query:    append(A,B,[1,2]) ?
answers:  A=[]      B=[1,2]
          A=[1]     B=[2]
          A=[1,2]   B=[]
```

Append

- **append([], L, L)**
- **append([H| L1], L2, [H| L3]) :- append(L1, L2, L3)**

- Example join [a, b, c] with [d, e].
  - [a, b, c] has the recursive structure [a| [b, c]].
  - Then the rule says:
  - IF [b,c] appends with [d, e] to form [b, c, d, e] THEN [a|[b, c]] appends with [d,e] to form [a|[b, c, d, e]]
  - i.e. [a, b, c] [a, b, c, d, e]

Expanding Prolog

- Parallelization:

  OR-parallelism: goal may unify with many different literals and implications in KB

  AND-parallelism: solve each conjunct in body of an implication in parallel

- Compilation: generate built-in theorem prover for different predicates in KB

- Optimization: for example through re-ordering

  e.g., “what is the income of the spouse of the president?”

  Income(s, i) $\wedge$ Married(s, p) $\wedge$ Occupation(p, President)

  faster if re-ordered as:

  Occupation(p, President) $\wedge$ Married(s, p) $\wedge$ Income(s, i)

Theorem provers

- Differ from logic programming languages in that:
  - accept full FOL
  - results independent of form in which KB entered

OTTER

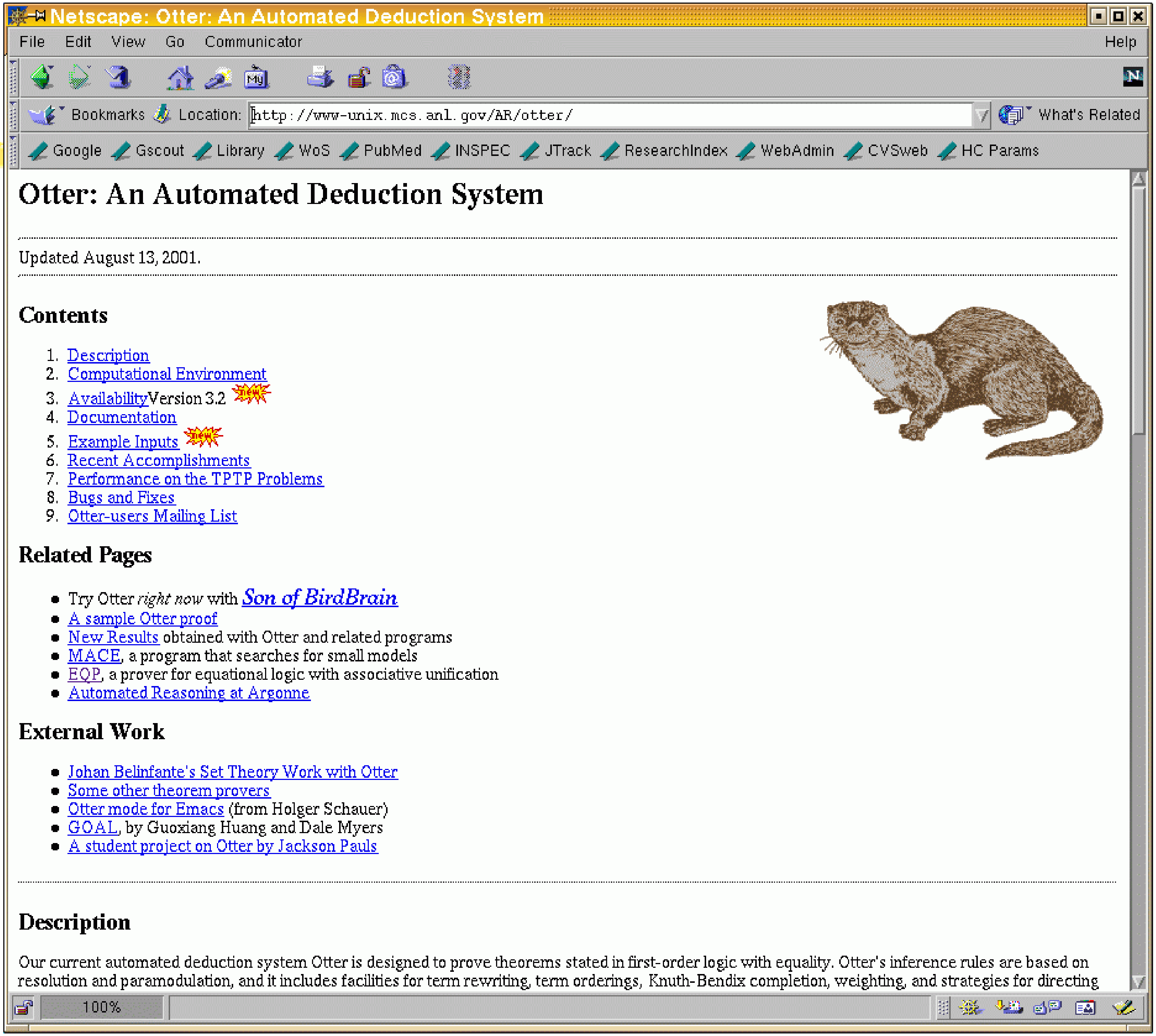
- Organized Techniques for Theorem Proving and Effective Research (McCune, 1992)

- Set of support (sos): set of clauses defining facts about problem
- Each resolution step: resolves member of sos against other axiom
- Usable axioms (outside sos): provide background knowledge about domain
- Rewrites (or demodulators): define canonical forms into which terms can be simplified. E.g., x+0=x
- Control strategy: defined by set of parameters and clauses. E.g., heuristic function to control search, filtering function to eliminate uninteresting subgoals.

OTTER

- Operation: resolve elements of sos against usable axioms
- Use best-first search: heuristic function measures “weight” of each clause (lighter weight preferred; thus in general weight correlated with size/difficulty)
- At each step: move lightest close in sos to usable list, and add to usable list consequences of resolving that close against usable list
- Halt: when refutation found or sos empty

Example



Otter: An Automated Deduction System

Updated August 13, 2001.

Contents

1. Description
2. Computational Environment
3. AvailabilityVersion 3.2 NEW
4. Documentation
5. Example Inputs NEW
6. Recent Accomplishments
7. Performance on the TPTP Problems
8. Bugs and Fixes
9. Otter-users Mailing List

Related Pages

- Try Otter *right now* with *Son of BirdBrain*
- A sample Otter proof
- New Results obtained with Otter and related programs
- MACE, a program that searches for small models
- EQP, a prover for equational logic with associative unification
- Automated Reasoning at Argonne

External Work

- Johan Belinfante's Set Theory Work with Otter
- Some other theorem provers
- Otter mode for Emacs (from Holger Schauer)
- GOAL, by Guoxiang Huang and Dale Myers
- A student project on Otter by Jackson Pauls

Description

Our current automated deduction system Otter is designed to prove theorems stated in first-order logic with equality. Otter's inference rules are based on resolution and paramodulation, and it includes facilities for term rewriting, term orderings, Knuth-Bendix completion, weighting, and strategies for directing

Example: Robbins Algebras Are Boolean

- The Robbins problem---are all Robbins algebras Boolean?---has been solved: Every Robbins algebra is Boolean. This theorem was proved automatically by EQP, a theorem proving program developed at Argonne National Laboratory

Example: Robbins Algebras Are Boolean

Historical Background

- In 1933, E. V. Huntington presented [1,2] the following basis for Boolean algebra:
- $x + y = y + x$. [commutativity]
- $(x + y) + z = x + (y + z)$. [associativity]
- $n(n(x) + y) + n(n(x) + n(y)) = x$. [Huntington equation]

- Shortly thereafter, Herbert Robbins conjectured that the Huntington equation can be replaced with a simpler one [5]:
- $n(n(x + y) + n(x + n(y))) = x$. [Robbins equation]
- Robbins and Huntington could not find a proof, and the problem was later studied by Tarski and his students

Example: Winker Conditions (1979)

- all x, n(n(x))=x
- ∃ 0 all x, x+0=x
- all x, x+x=x
- 1st: ∃ C ∃ D, C+D=C
- 2nd: ∃ C ∃ D, n(C+D)=n(C)

Example: Otter: October 10, 1996

- n(n(n(y)+x)+n(x+y)) = x. [Robbins equation]
- n(x+y) != n(x). [denial of 2nd Winker condition]

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```
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```

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```
n(n(x + y) + n(x + n(y))) = x.        [Robbins equation]
```

Robbins and Huntington could not find a proof, and the problem was later studied by Tarski and his students [6].

Given to the system

```
Searching ...

Success, in 1.28 seconds!

---------------- PROOF ----------------

1         n(n(A)+B)+n(n(A)+n(B))!=A.                      []
2         x=x.                                            []
3         x+y=y+x.                                        []
5,4       (x+y)+z=x+ (y+z).                               []
6         n(n(x+y)+n(x+n(y)))=x.                          []
8         x+x=x.                                          []

10        n(n(A)+n(B))+n(n(A)+B)!=A.                      [para_from,3,1]
13        x+ (x+y)=x+y.                                   [para_into,4,8,flip.1]
15        x+ (y+z)=y+ (x+z).                              [para_into,4,3,demod,5]
23,22     x+ (y+x)=x+y.                                   [para_into,13,3]
26        n(n(x)+n(x+n(x)))=x.                            [para_into,6,8]
36        n(n(n(x)+x)+n(n(x)))=n(x).                      [para_into,6,8]
42        n(n(x+n(y))+n(x+y))=x.                          [para_into,6,3]
52        x+ (y+z)=x+ (z+y).                              [para_into,15,3,demod,5]
81,80     n(n(x+n(x))+n(x))=x.                            [para_into,26,3]
82        n(n(n(x)+x)+x)=n(x).                            [para_from,26,6,demod,23]
125       n(n(n(x+n(x))+ (n(x)+x))+x)=n(x+n(x))+n(x).  [para_into,80,80,demod,5,81]
139       n(n(n(x+n(x))+x)+x)=n(x+n(x)).                 [para_from,80,6]
166,165   n(n(x+n(x))+x)=n(x).                            [para_into,82,3]
180,179   n(n(x)+x)=n(x+n(x)).                            [back_demod,139,demod,166]
195       n(n(x+n(x))+n(n(x)))=n(x).                      [back_demod,36,demod,180]
197       n(n(x+ (n(x)+n(x+n(x))))+ (n(x+n(x))+x))=n(x).  [para_into,165,165,demod,5,180,5,166]
206,205   n(n(x+ (n(x)+n(x+n(x))))+n(x))=n(x+n(x))+x.  [para_from,165,80,demod,166,5,180,5]
223,222   n(n(x+y)+ (y+x))=n(x+ (y+n(x+y))).             [para_into,179,52,demod,5]
231,230   n(n(x+ (n(x)+n(x+n(x))))+x)=n(x+n(x))+n(x).  [back_demod,125,demod,223]
564,563   n(x+n(x))+x=x.                                  [para_into,195,80,demod,5,223,81,206,81]
582,581   n(x+n(x))+n(x)=n(x).                            [back_demod,197,demod,564,231]
586,585   n(n(x))=x.                                      [back_demod,80,demod,582]
606,605   n(x+n(y))+n(x+y)=n(x).                          [para_into,585,42,flip.1]
621       A!=A.                                           [back_demod,10,demod,606,586]
622       $F.                                             [binary,621,2]

------------ end of proof -------------
```

Forward-chaining production systems

- Prolog & other programming languages: rely on backward-chaining

  (I.e., given a query, find substitutions that satisfy it)

- Forward-chaining systems: infer everything that can be inferred from KB each time new sentence is TELL'ed

- Appropriate for agent design: as new percepts come in, forward-chaining returns best action

Implementation

- One possible approach: use a theorem prover, using resolution to forward-chain over KB

- More restricted systems can be more efficient.

- Typical components:
  - KB called "working memory" (positive literals, no variables)
  - rule memory (set of inference rules in form

    $p1 \wedge p2 \wedge \ldots \Rightarrow act1 \wedge act2 \wedge \ldots$
  - at each cycle: find rules whose premises satisfied by working memory (match phase)
  - decide which should be executed (conflict resolution phase)
  - execute actions of chosen rule (act phase)

Match phase

- Unification can do it, but inefficient

- Rete algorithm (used in OPS-5 system): example

rule memory:

$A(x) \wedge B(x) \wedge C(y) \Rightarrow$ add $D(x)$

$A(x) \wedge B(y) \wedge D(x) \Rightarrow$ add $E(x)$

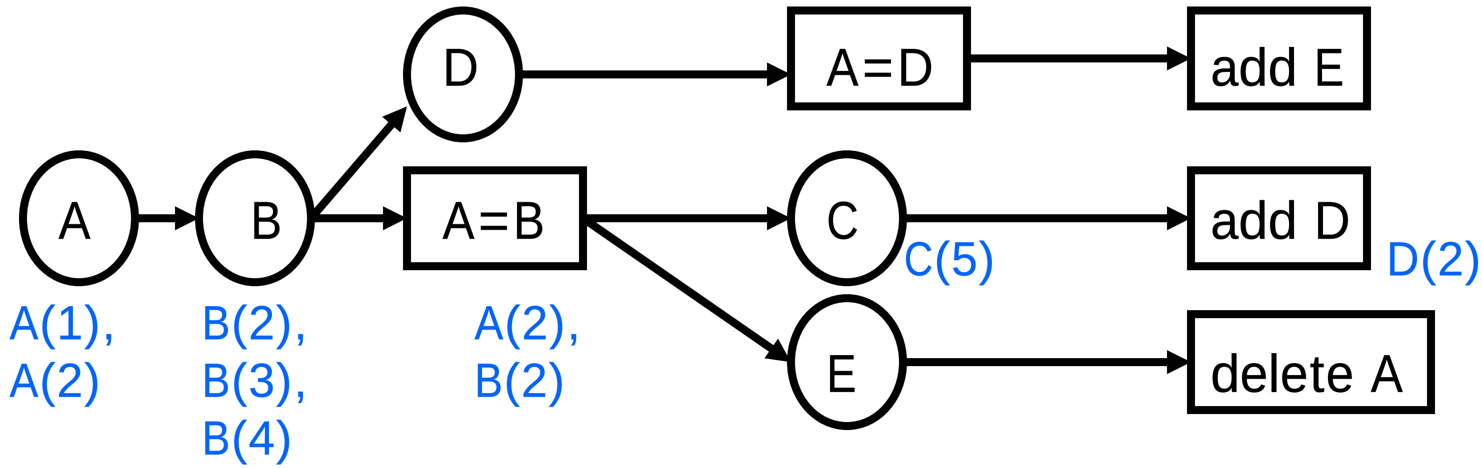
$A(x) \wedge B(x) \wedge E(x) \Rightarrow$ delete $A(x)$

working memory:

$\{A(1), A(2), B(2), B(3), B(4), C(5)\}$

- Build Rete network from rule memory, then pass working memory through it

Rete network



Circular nodes: fetches to WM; rectangular nodes: unifications

$A(x) \wedge B(x) \wedge C(y) \Rightarrow$ add $D(x)$

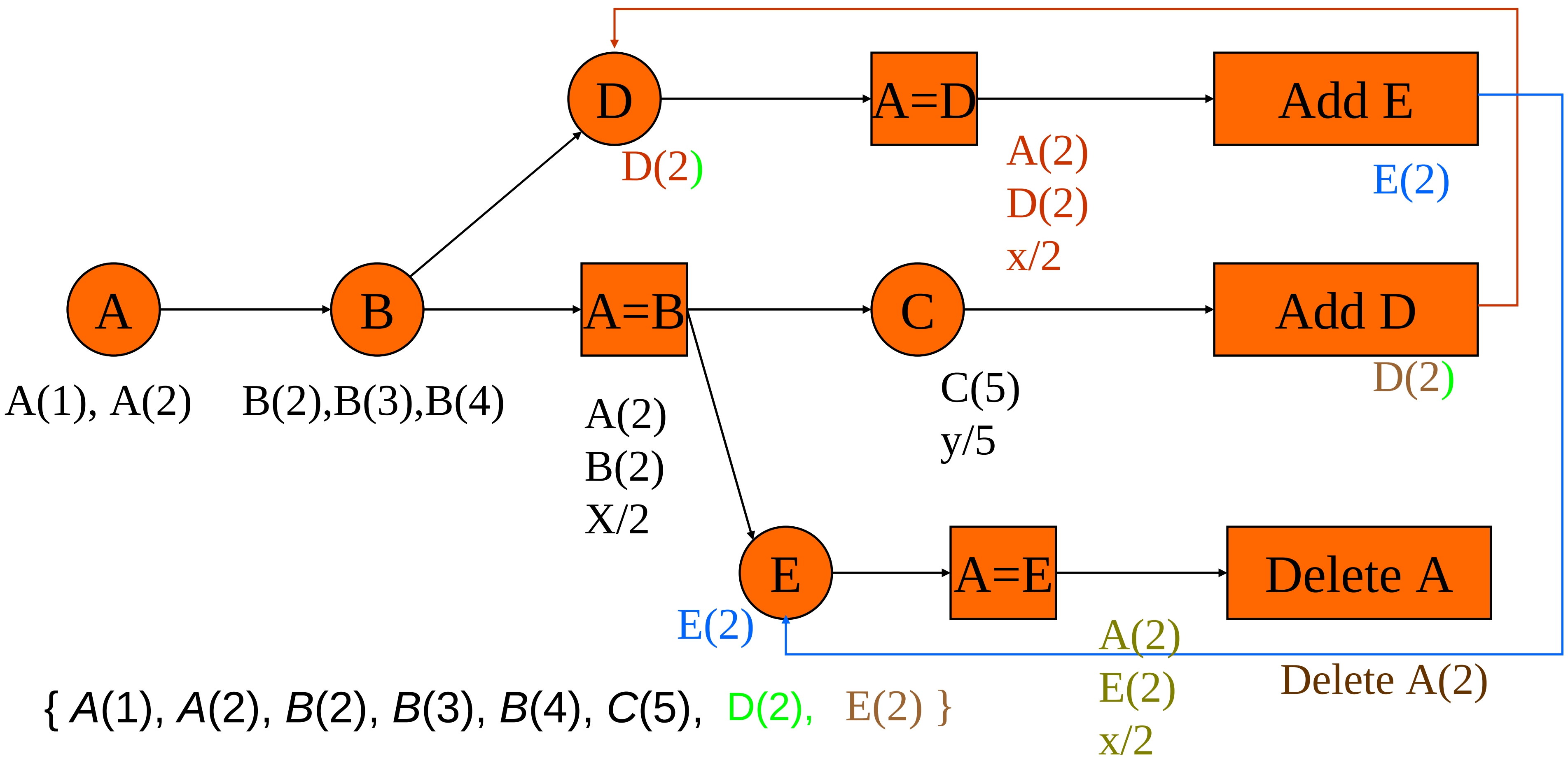
$A(x) \wedge B(y) \wedge D(x) \Rightarrow$ add $E(x)$

$A(x) \wedge B(x) \wedge E(x) \Rightarrow$ delete $A(x)$

{A(1), A(2), B(2), B(3), B(4), C(5)}

Rete match

$A(x) \wedge B(x) \wedge C(y) \Rightarrow$ add $D(x)$
$A(x) \wedge B(y) \wedge D(x) \Rightarrow$ add $E(x)$
$A(x) \wedge B(x) \wedge E(x) \Rightarrow$ delete $A(x)$



{ A(1), A(2), B(2), B(3), B(4), C(5), D(2), E(2) }

Advantages of Rete networks

- Share common parts of rules
- Eliminate duplication over time (since for most production systems only a few rules change at each time step)

Conflict resolution phase

- one strategy: execute all actions for all satisfied rules

- or, treat them as suggestions and use conflict resolution to pick one action.

- Strategies:

  - no duplication (do not execute twice same rule on same args)

  - regency (prefer rules involving recently created WM elements)

  - specificity (prefer more specific rules)

  - operation priority (rank actions by priority and pick highest)

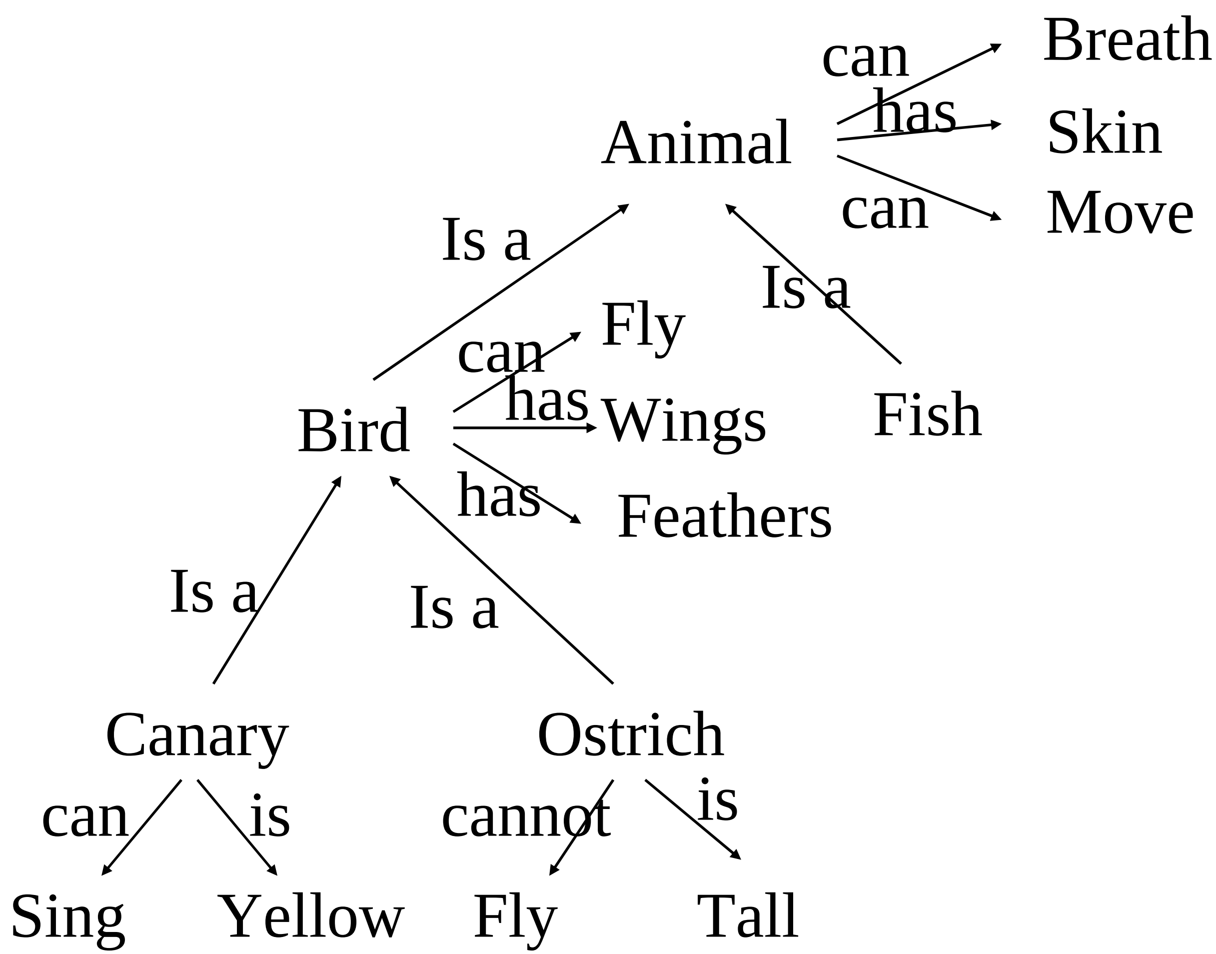
Frame systems & semantic networks

- Other notation for logic; equivalent to sentence notation
- Focus on categories and relations between them (remember ontologies)
- e.g., Cats $\xrightarrow{\textit{Subset}}$ Mammals

Syntax and Semantics

Link Type	Semantics
$A \xrightarrow{\text{Subset}} B$	$A \subset B$
$A \xrightarrow{\text{Member}} B$	$A \in B$
$A \xrightarrow{R} B$	$R(A,B)$
$A \xrightarrow{\boxed{R}} B$	$\forall x\ x \in A \Rightarrow R(x,y)$
$A \xrightarrow{\boxed{\boxed{R}}} B$	$\forall x\ \exists y\ x \in A \Rightarrow y \in B \wedge R(x,y)$

Semantic Network Representation



Semantic network link types

Link type	Semantics	Example
$A \xrightarrow{\mathit{Subset}} B$	$A \subset B$	$\text{Cats} \xrightarrow{\mathit{Subset}} \text{Mammals}$
$A \xrightarrow{\mathit{Member}} B$	$A \in B$	$\text{Bill} \xrightarrow{\mathit{Member}} \text{Cats}$
$A \xrightarrow{R} B$	$R(A, B)$	$\text{Bill} \xrightarrow{\mathit{Age}} 12$
$A \xrightarrow{\boxed{R}} B$	$\forall x \quad x \in A \Rightarrow R(x, B)$	$\text{Birds} \xrightarrow{\boxed{\mathit{Legs}}} 2$
$A \xrightarrow{\boxed{\boxed{R}}} B$	$\forall x\ \exists y\ x \in A \Rightarrow y \in B \wedge R(x, y)$	$\text{Birds} \xrightarrow{\boxed{\boxed{\mathit{Parent}}}} \text{Birds}$

Description logics

- FOL: focus on objects

- Description logics: focus on categories and their definitions

- Principal inference tasks:
  - subsumption: is one category subset of another?
  - classification: object belings to category?

CLASSIC

- And(concept, ...)
- All(RoleName, Concept)
- AtLeast(Integer, RoleName)
- AtMost(Integer, RolaName)
- Fills(RoleName, IndividualName, ...)
- SameAs(Path, Path)
- OneOf(IndividualName, ...)

e.g., Bachelor = And(Unmarried, Adult, Male)