This time: Fuzzy Logic and Fuzzy Inference

- Why use fuzzy logic?
- Tipping example
- Fuzzy set theory
- Fuzzy inference

What is fuzzy logic?

- A super set of Boolean logic
- Builds upon fuzzy set theory
- Graded truth. Truth values between True and False. Not everything is either/or, true/false, black/white, on/off etc.
- Grades of membership. Class of tall men, class of far cities, class of expensive things, etc.
- Lotfi Zadeh, UC/Berkely 1965. Introduced **FL to model uncertainty in natural language**. *Tall, far, nice, large, hot, ...*
- Reasoning using linguistic terms. Natural to express expert knowledge.

  If the weather is cold then wear warm clothing

Why use fuzzy logic?

Pros:

- Conceptually easy to understand w/ “natural” maths
- Tolerant of imprecise data
- Universal approximation: can model arbitrary nonlinear functions
- Intuitive
- Based on linguistic terms
- Convenient way to express expert and common sense knowledge

Cons:

- Not a cure-all
- Crisp/precise models can be more efficient and even convenient
- Other approaches might be formally verified to work

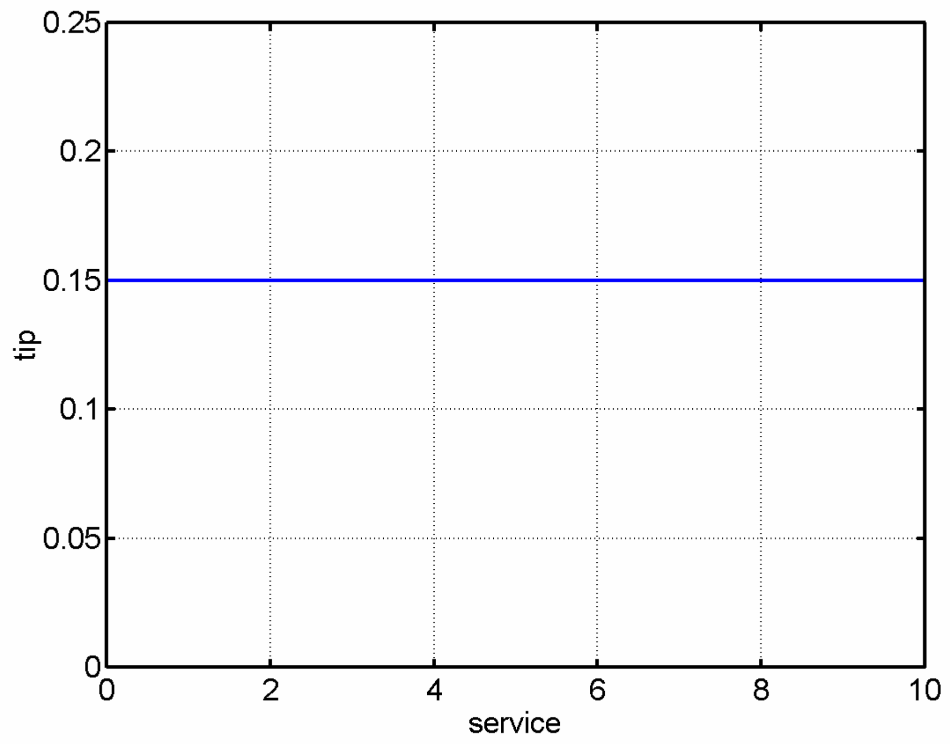
Tipping example

- **The Basic Tipping Problem:** Given a number between 0 and 10 that represents the quality of service at a restaurant what should the tip be?

  Cultural footnote: An average tip for a meal in the U.S. is 15%, which may vary depending on the quality of the service provided.

Tipping example: The non-fuzzy approach

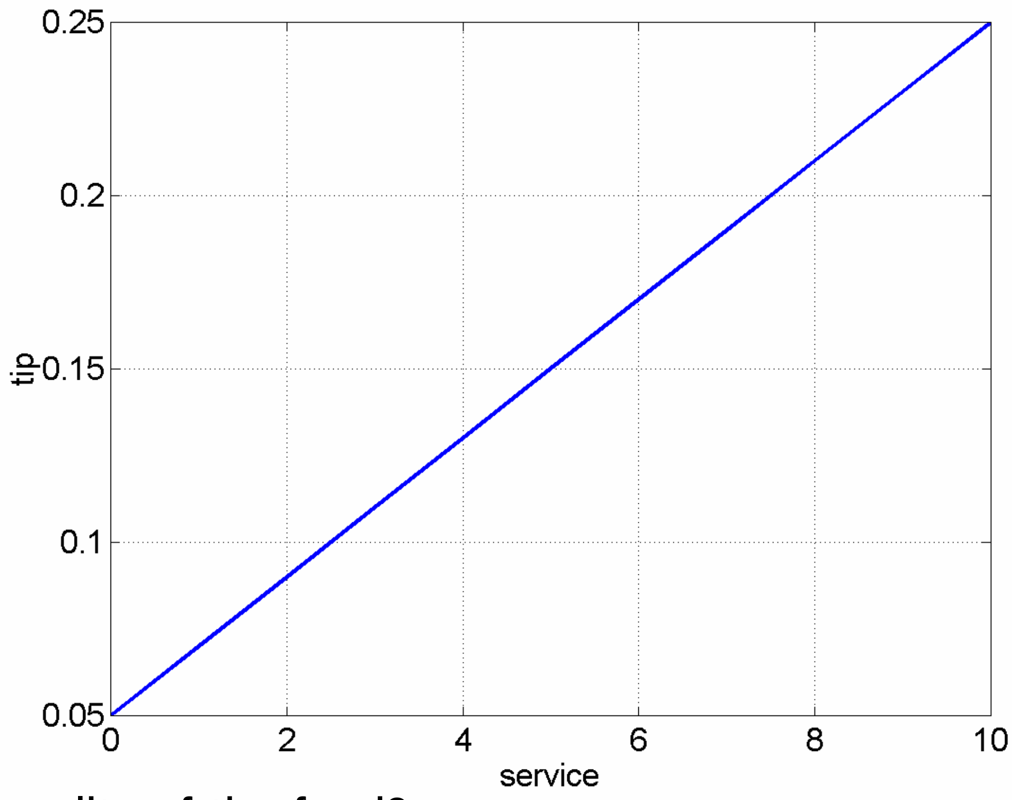
- Tip = 15% of total bill



- What about quality of service?

Tipping example: The non-fuzzy approach

- Tip = linearly proportional to service from 5% to 25%
  tip = 0.20/10*service+0.05



- What about quality of the food?

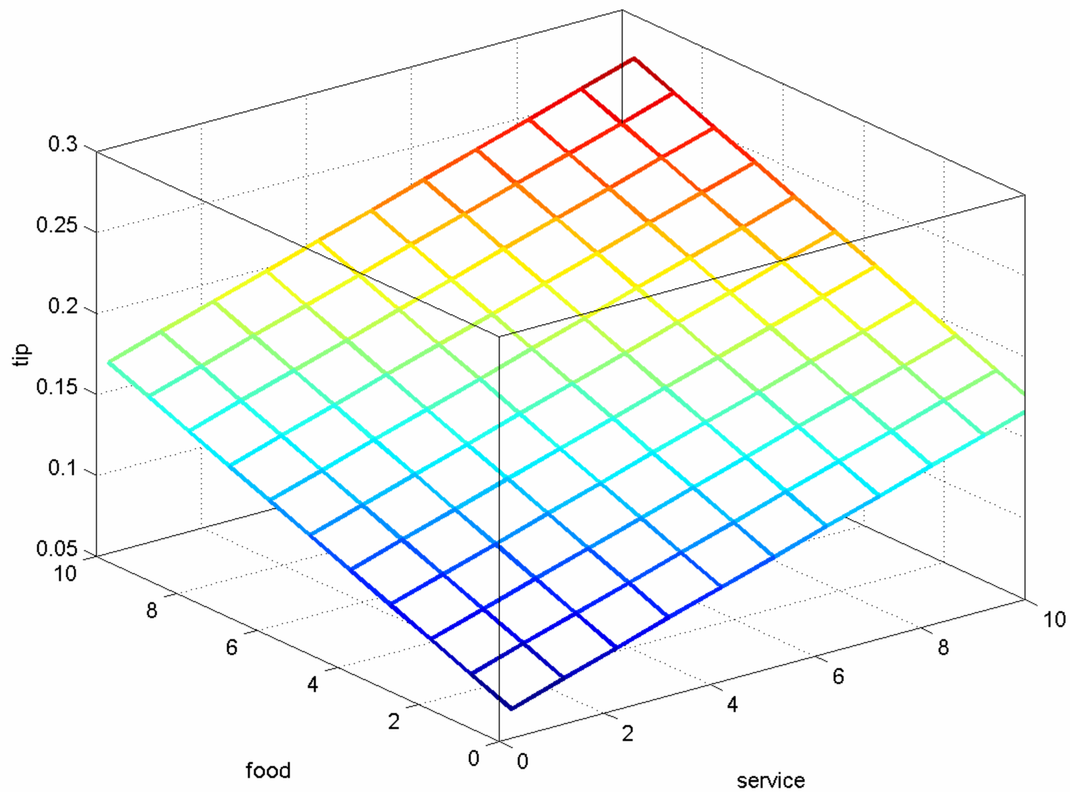
Tipping example: Extended

- **The Extended Tipping Problem:** Given a number between 0 and 10 that represents the quality of service <u>and the quality of the food</u>, at a restaurant, what should the tip be?

  How will this affect our tipping formula?

Tipping example: The non-fuzzy approach

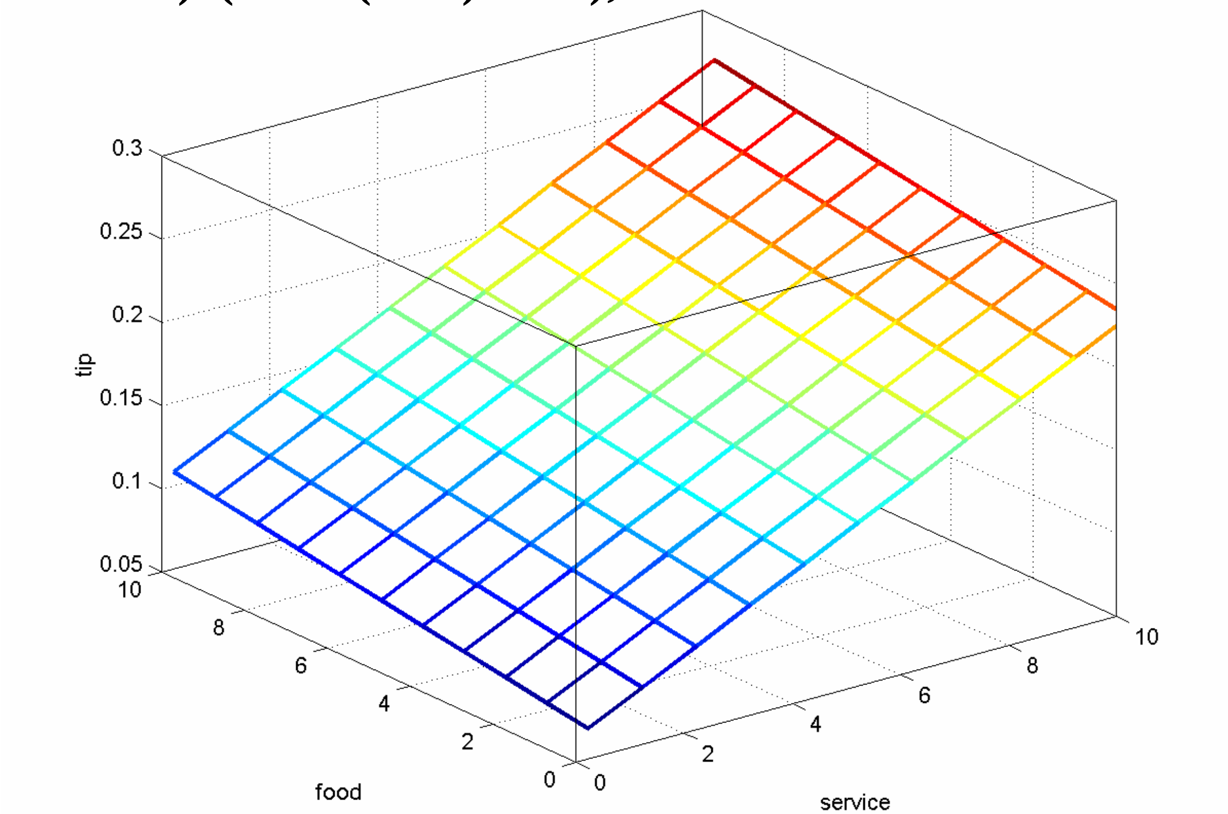
- Tip = 0.20/20*(service+food)+0.05



- We want service to be more important than food quality. E.g., 80% for service and 20% for food.

Tipping example: The non-fuzzy approach

- Tip = **servRatio*(.2/10*(service)+.05) + (1-servRatio)*(.2/10*(food)+0.05);** **servRatio = 80%**



- Seems too linear. Want 15% tip in general and deviation only for exceptionally good or bad service.

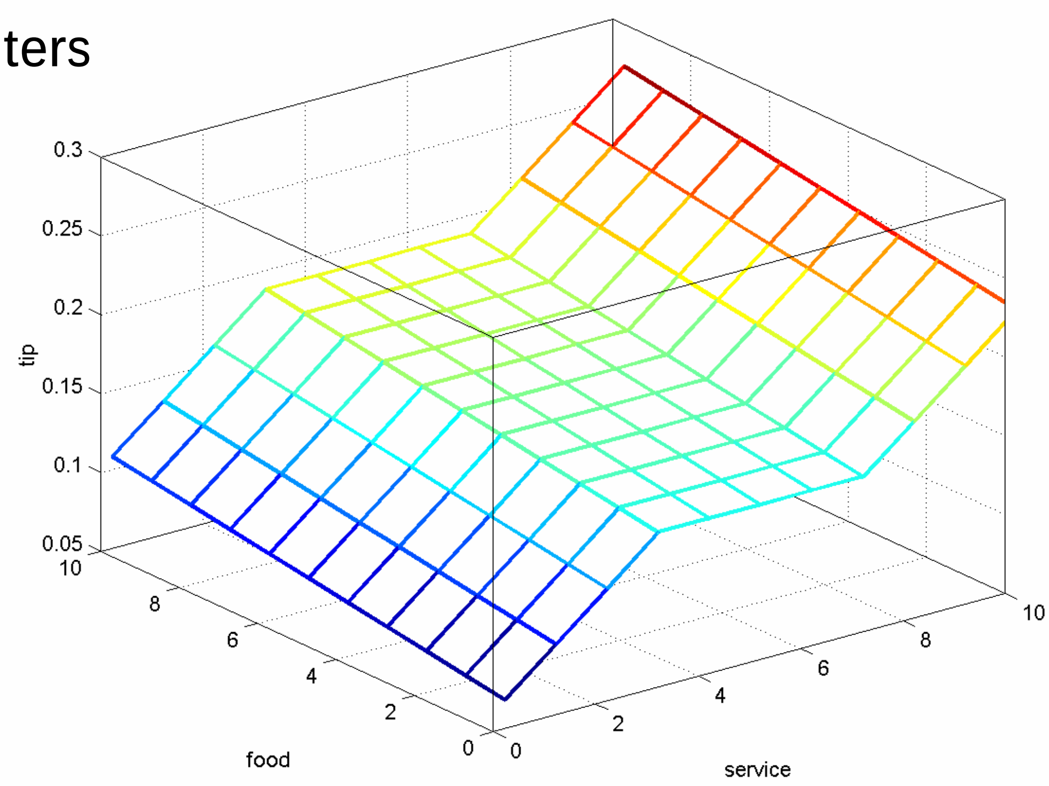
Tipping example: The non-fuzzy approach

```
if service < 3,
  tip(f+1,s+1) = servRatio*(.1/3*(s)+.05) + ...
                 (1-servRatio)*(.2/10*(f)+0.05);
elseif s < 7,
  tip(f+1,s+1) = servRatio*(.15) + ...
                 (1-servRatio)*(.2/10*(f)+0.05);
else,
  tip(f+1,s+1) = servRatio*(.1/3*(s-7)+.15) + ...
                 (1-servRatio)*(.2/10*(f)+0.05);
end;
```

Tipping example: The non-fuzzy approach

Nice plot <u>but</u>

- ‘Complicated’ function
- Not easy to modify
- Not intuitive
- Many hard-coded parameters
- Not easy to understand



Tipping problem: the fuzzy approach

What we want to express is:

1. *If service is poor then tip is cheap*
2. *If service is good the tip is average*
3. *If service is excellent then tip is generous*
4. *If food is rancid then tip is cheap*
5. *If food is delicious then tip is generous*

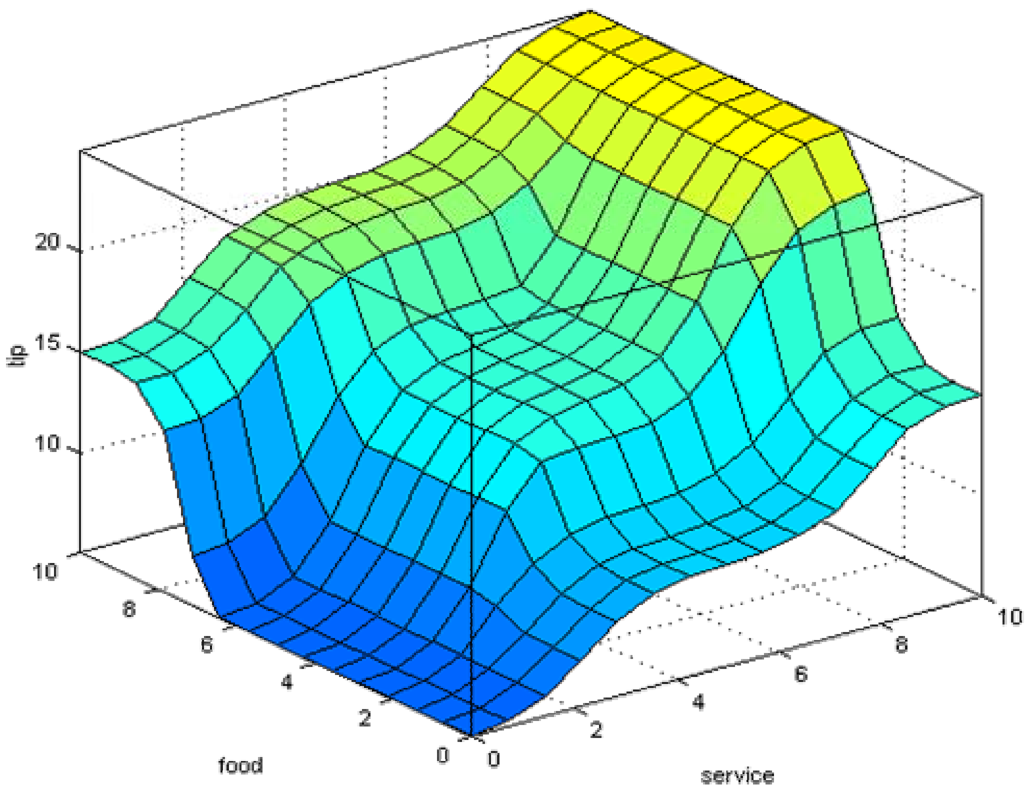
or

1. *If service is poor or the food is rancid then tip is cheap*
2. *If service is good then tip is average*
3. *If service is excellent or food is delicious then tip is generous*

We have just defined the rules for a fuzzy logic system.

Tipping problem: fuzzy solution

Decision function generated using the 3 rules.



Tipping problem: fuzzy solution

- Before we have a fuzzy solution we need to find out

a) how to define terms such as *poor, delicious*, *cheap, generous etc.*
b) how to combine terms using AND, OR and other connectives
c) how to combine all the rules into one final output

Fuzzy sets

- **Boolean / Crisp set A** is a mapping for the elements of S to the set {0, 1}, i.e., A: S → {0, 1}
- ***Characteristic function:***

$$\mu_A(x) = \begin{cases} 1 & \text{if x is an element of set A} \\ 0 & \text{if x is not an element of set A} \end{cases}$$

- **Fuzzy set F** is a mapping for the elements of S to the interval [0, 1], i.e., F: S → [0, 1]
- Characteristic function: $0 \le \mu_F(x) \le 1$
- 1 means full membership, 0 means no membership and anything in between, e.g., 0.5 is called **graded membership**

Example: Crisp set Tall

- Fuzzy sets and concepts are commonly used in natural language

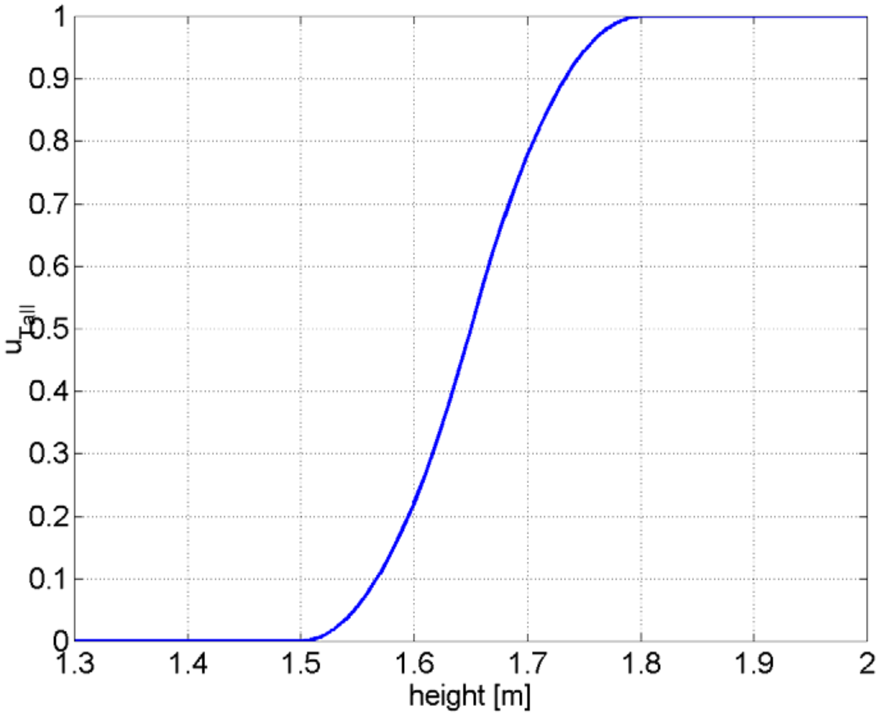
  *John is **tall***
  *Dan is **smart***
  *Alex is **happy***
  *The class is **hot***

- E.g., the crisp set ***Tall*** can be defined as $\{x \mid \text{height } x > 1.8 \text{ meters}\}$
  But what about a person with a height = 1.79 meters?
  What about 1.78 meters?
  …
  What about 1.52 meters?

Example: Fuzzy set Tall

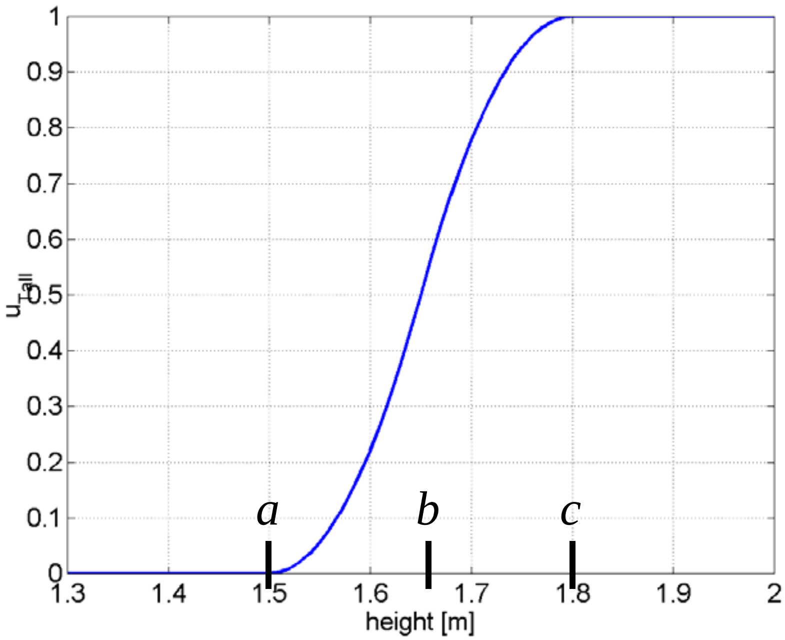
- In a fuzzy set a person with a height of 1.8 meters would be considered tall to a **high degree**
  A person with a height of 1.7 meters would be considered tall to a lesser degree etc.

- The function can change for basketball players, Danes, women, children etc.



Membership functions: S-function

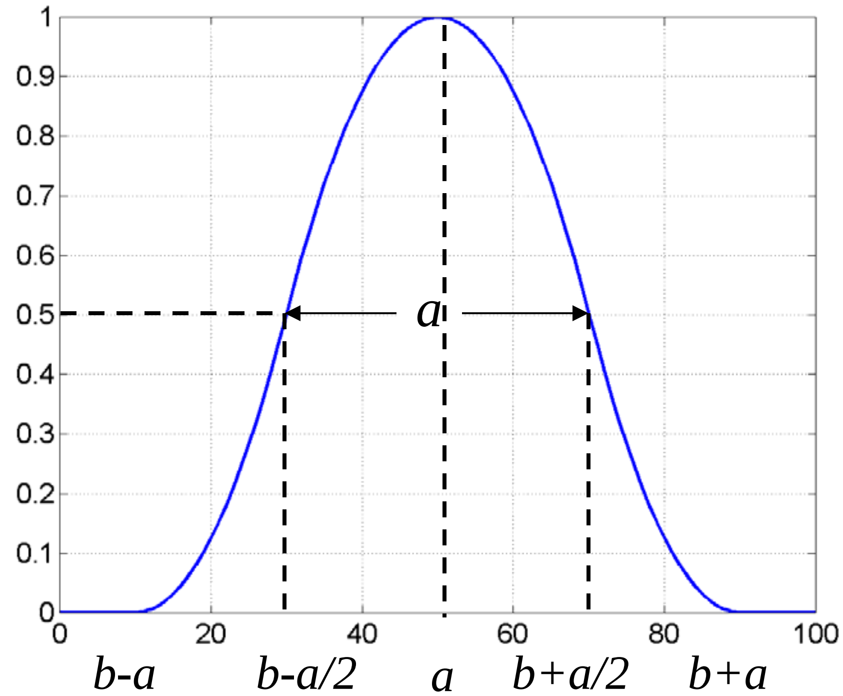
- The S-function can be used to define fuzzy sets
- $S(x, a, b, c) =$
  - 0 for $x \leq a$
  - $2(x\text{-a/c-a})^2$ for $a \leq x \leq b$
  - $1 - 2(x\text{-}c/c\text{-}a)^2$ for $b \leq x \leq c$
  - 1 for $x \geq c$



Membership functions: Π–Function

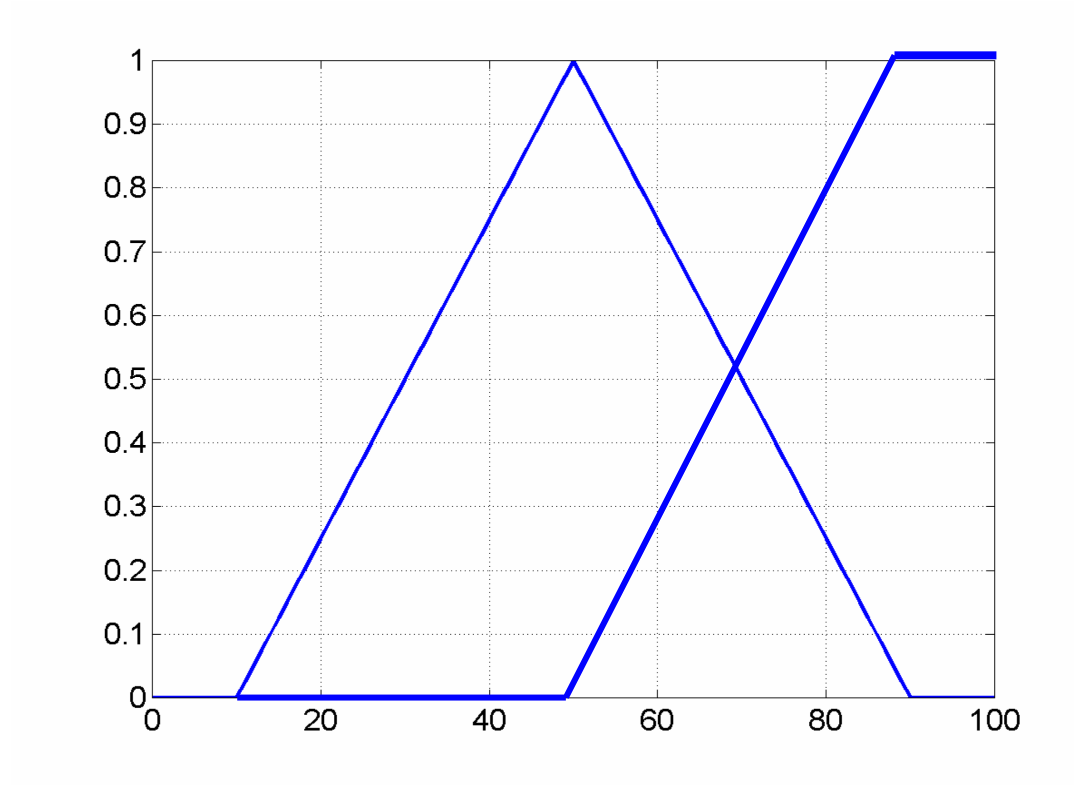
- $\Pi(x, a, b) =$
  - $S(x, b\text{-}a, b\text{-}a/2, b)$ for $x \le b$
  - $1 - S(x, b, b+a/2, a+b)$ for $x \ge b$

E.g., ***close*** (to a)

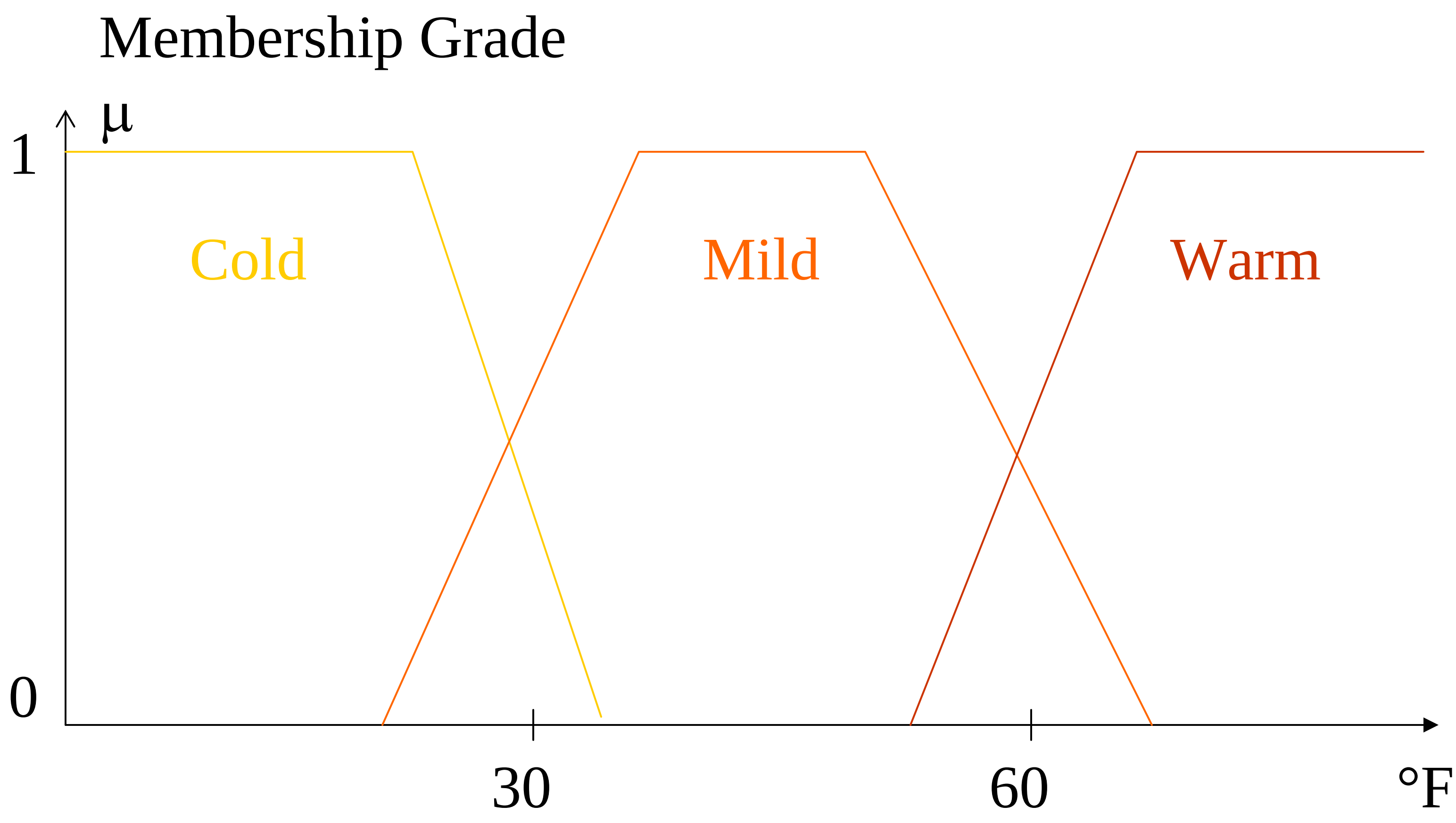


Simple membership functions

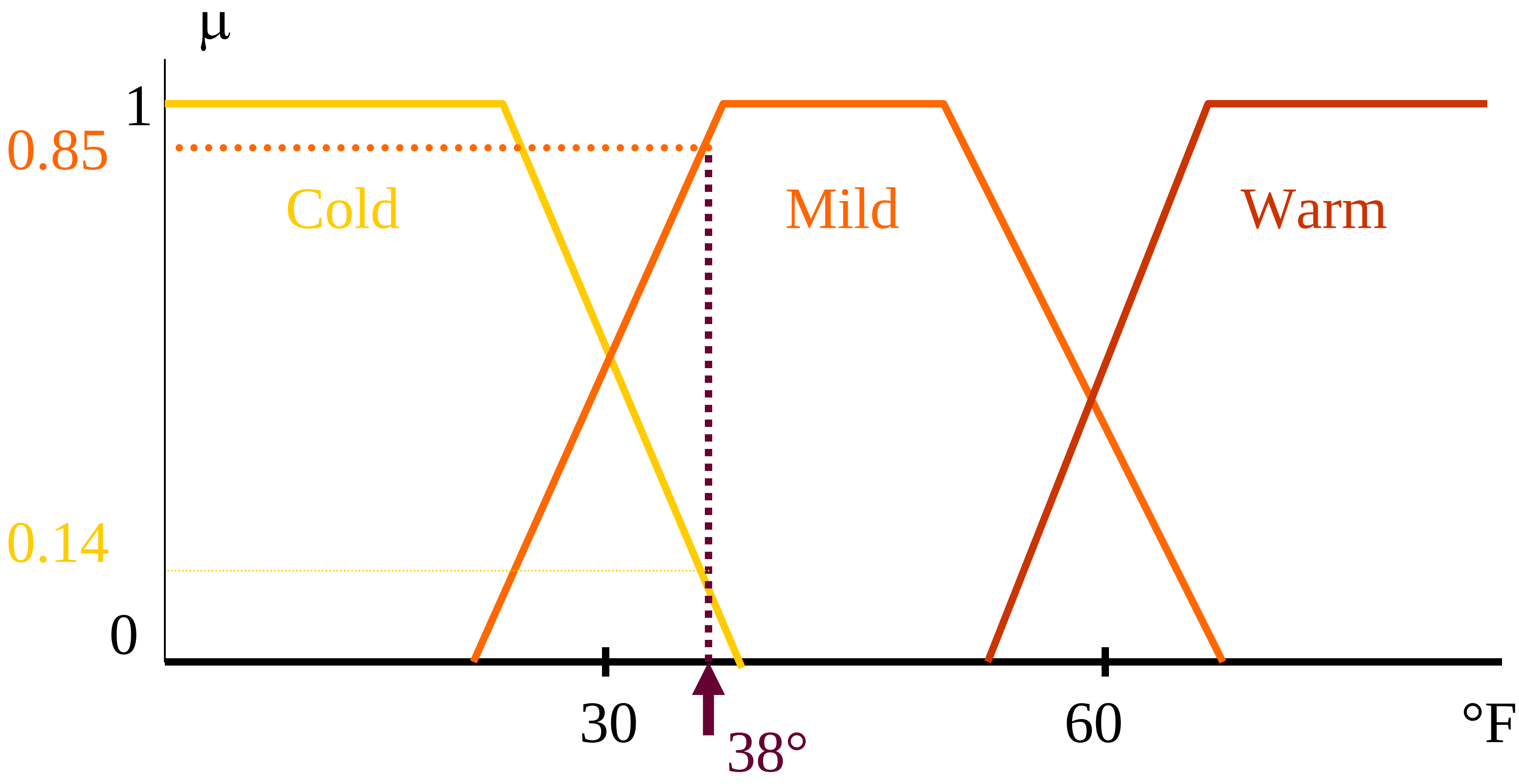
- Piecewise linear: triangular etc.
- Easier to represent and calculate ⇒ saves computation



Fuzzy Sets



Observation



Other representations of fuzzy sets

- A finite set of elements:

  $F = \mu_1/x_1 + \mu_2/x_2 + \ldots \mu_n/x_n$

  + means (Boolean) set union

- For example:

  TALL = {0/1.0, 0/1.2, 0/1.4, 0.2/1.6, 0.8/1.7, 1.0/1.8}

Fuzzy set operators

- Equality

  $A = B$

  $\mu_A(x) = \mu_B(x)$ for all $x \in X$
- Complement

  A'

  $\mu_{A'}(x) = 1 - \mu_A(x)$ for all $x \in X$
- Containment

  $A \subseteq B$

  $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$
- Union

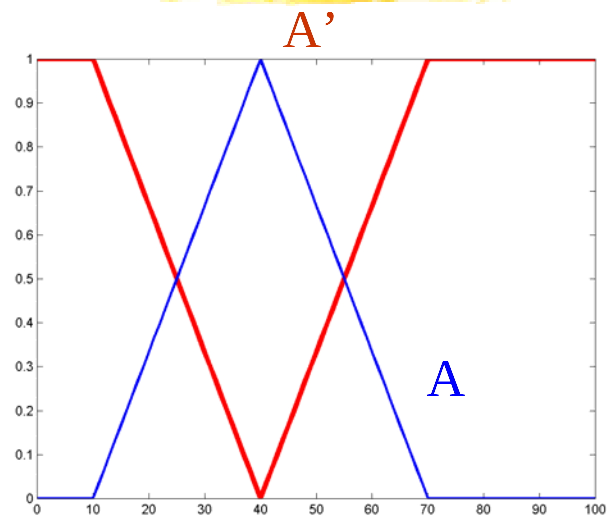
  $A \cup B$

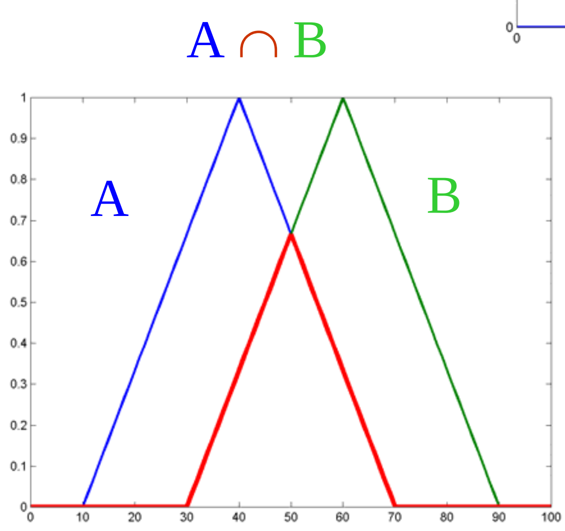
  $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$ for all $x \in X$
- Intersection

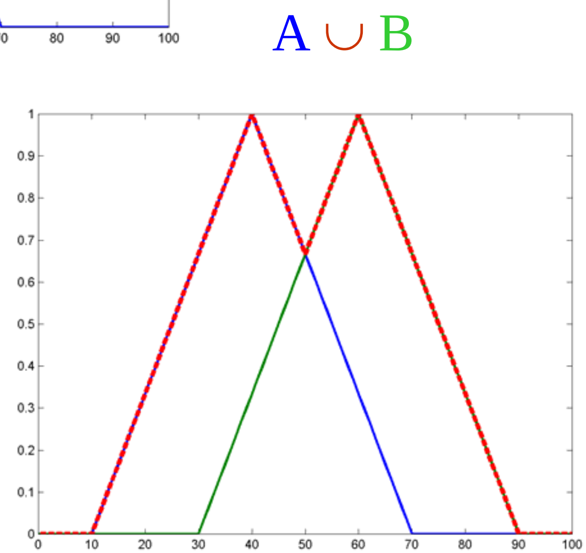
  $A \cap B$

  $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$ for all $x \in X$

Example fuzzy set operations



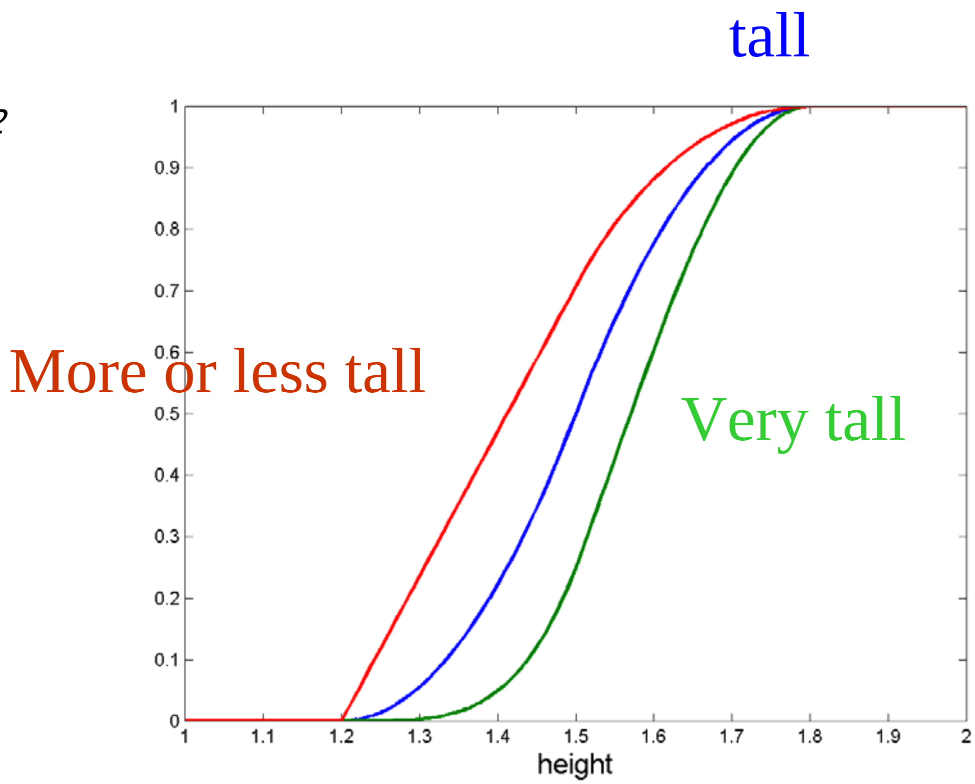




Linguistic Hedges

- Modifying the meaning of a fuzzy set using hedges such as *very, more or less, slightly, etc.*

- *Very* $F = F^2$
- *More or less* $F = F^{1/2}$
- *etc.*



Fuzzy relations

- A fuzzy relation for N sets is defined as an extension of the crisp relation to include the membership grade.

  R = $\{\mu_R(x_1, x_2, \ldots x_N)/(x_1, x_2, \ldots x_N) \mid x_i \in X, i=1, \ldots N\}$

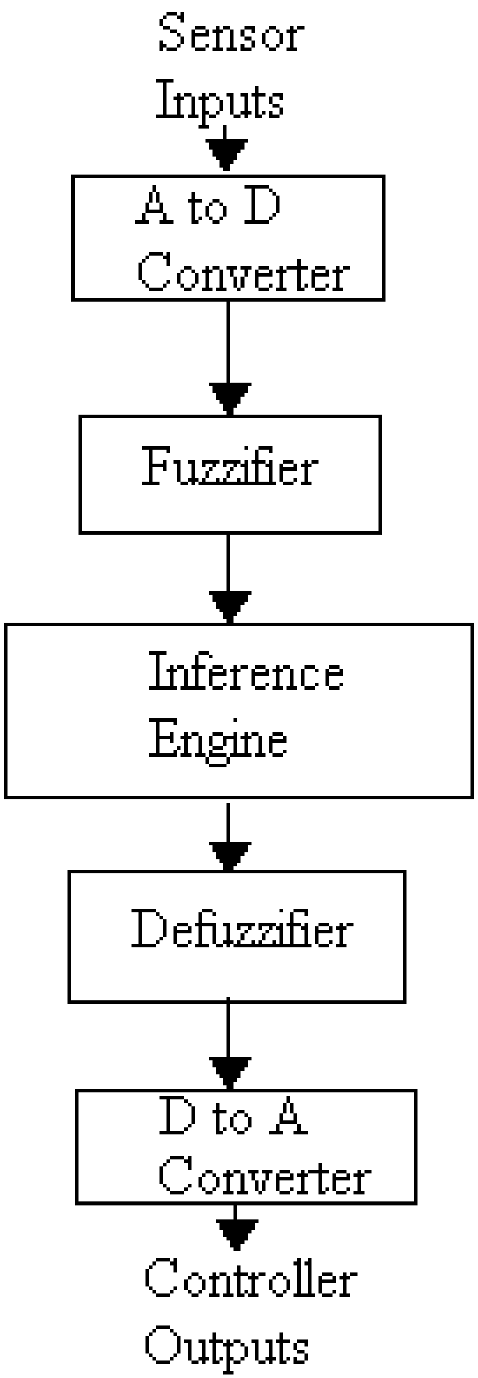
  which associates the membership grade, μ_R , of each tuple.

- E.g.

  Friend = {0.9/(Manos, Nacho), 0.1/(Manos, Dan),
  0.8/(Alex, Mike), 0.3/(Alex, John)}

Fuzzy inference

- Fuzzy logical operations
- Fuzzy rules
- Fuzzification
- Implication
- Aggregation
- Defuzzification



Fuzzy logical operations

- AND, OR, NOT, etc.

- **NOT** A = A' = 1 - $\mu_A(x)$
- A **AND** B = $A \cap B$ = $\min(\mu_A(x), \mu_B(x))$
- A **OR** B = $A \cup B$ = $\max(\mu_A(x), \mu_B(x))$

From the following truth tables it is seen that fuzzy logic is a **superset** of Boolean logic.

min(A,B)

A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

max(A,B)

A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

1-A

A	not A
0	1
1	0

If-Then Rules

- Use fuzzy sets and fuzzy operators as the **subjects** and **verbs** of fuzzy logic to form rules.

  if x is A then y is B

  where A and B are linguistic terms defined by fuzzy sets on the sets X and Y respectively.

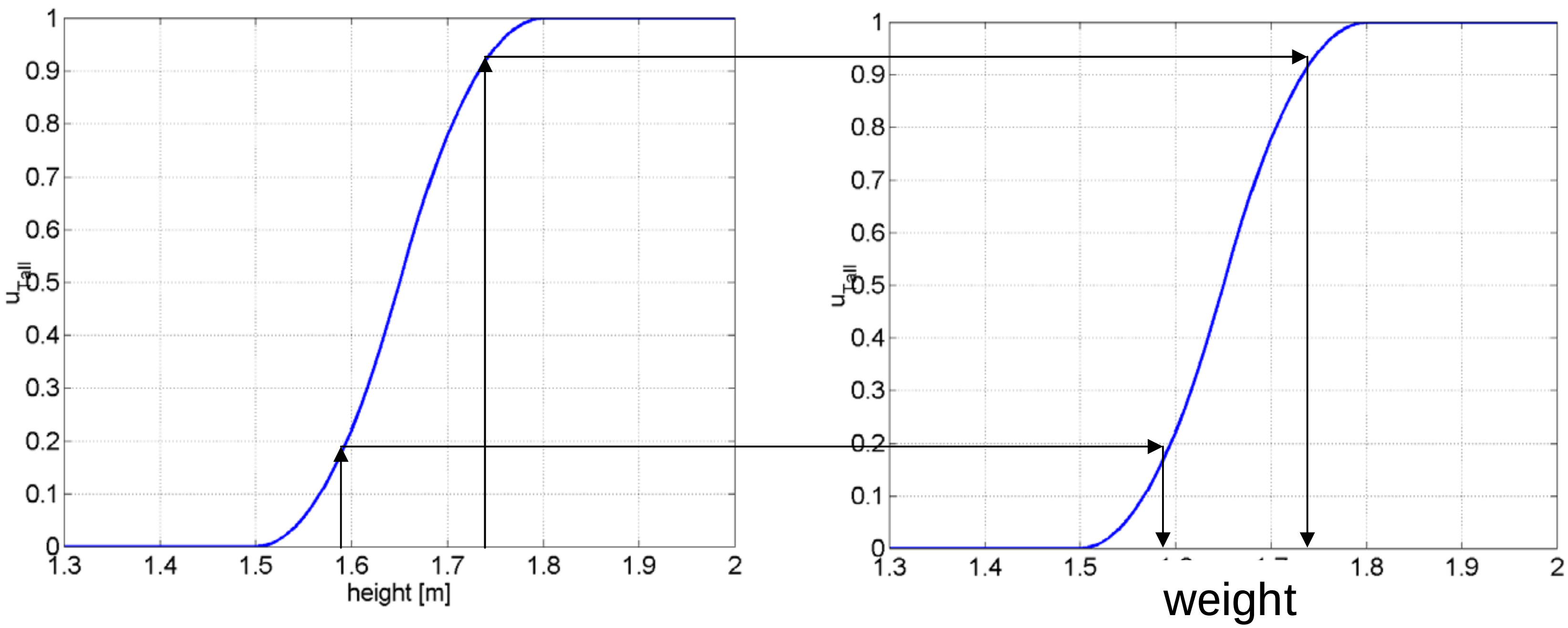
  This reads

  if <u>x == A</u> then <u>y = B</u>

Example:

- IF height is Tall THEN weight is Heavy

Tall men Heavy men



Example

- If it is hot, turn on the air conditioner
  - Determine if the current temp. belongs to the hot fuzzy set
  - If so, then turn on the air conditioner until it goes to the warm fuzzy set

Evaluation of fuzzy rules

- In Boolean logic: $p \Rightarrow q$
  if p is true then q is true

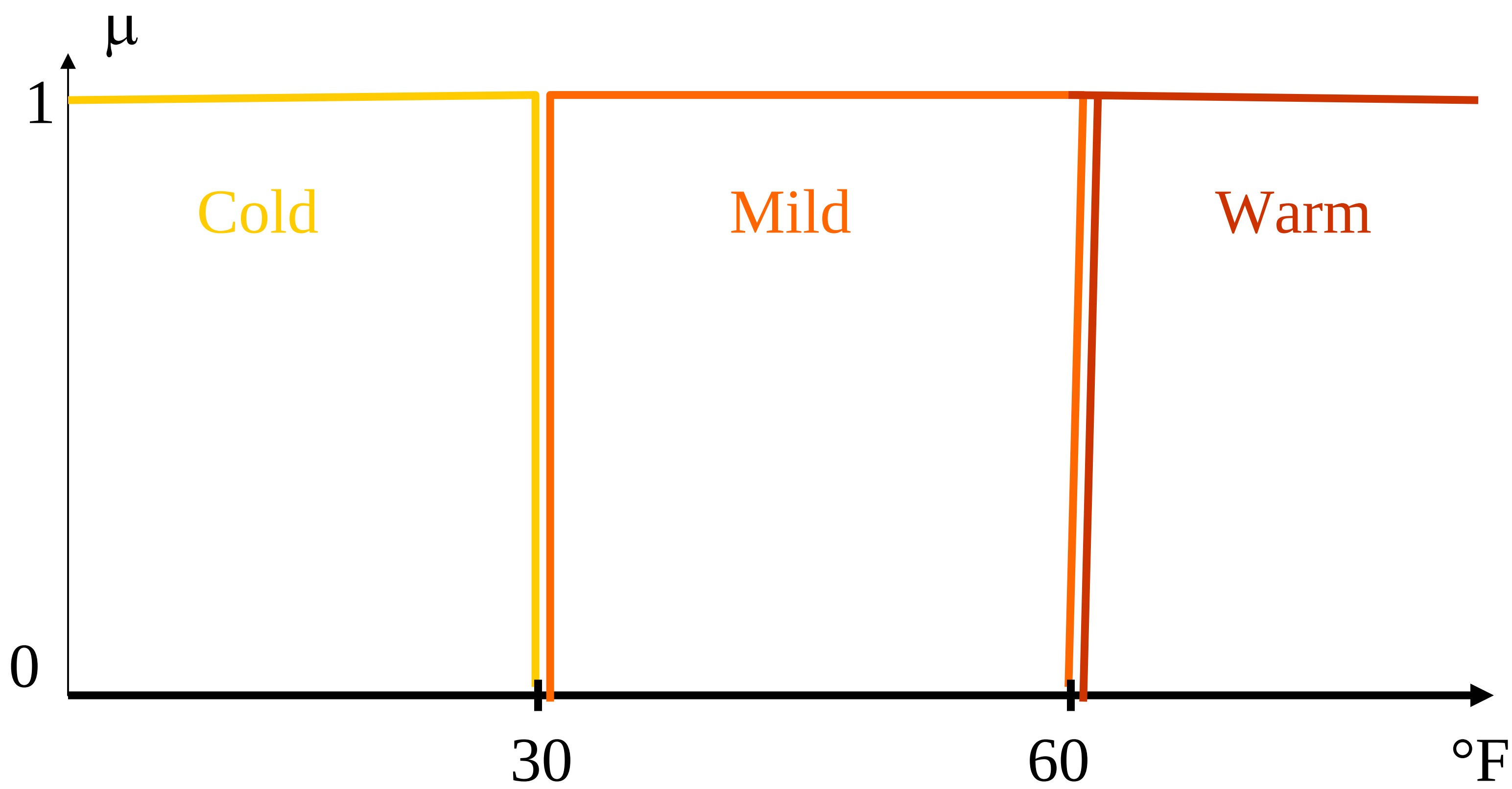
- In fuzzy logic: $p \Rightarrow q$
  if p is true to some degree then q is true to some degree.

  0.5p => 0.5q (partial premise implies partially)

- How?

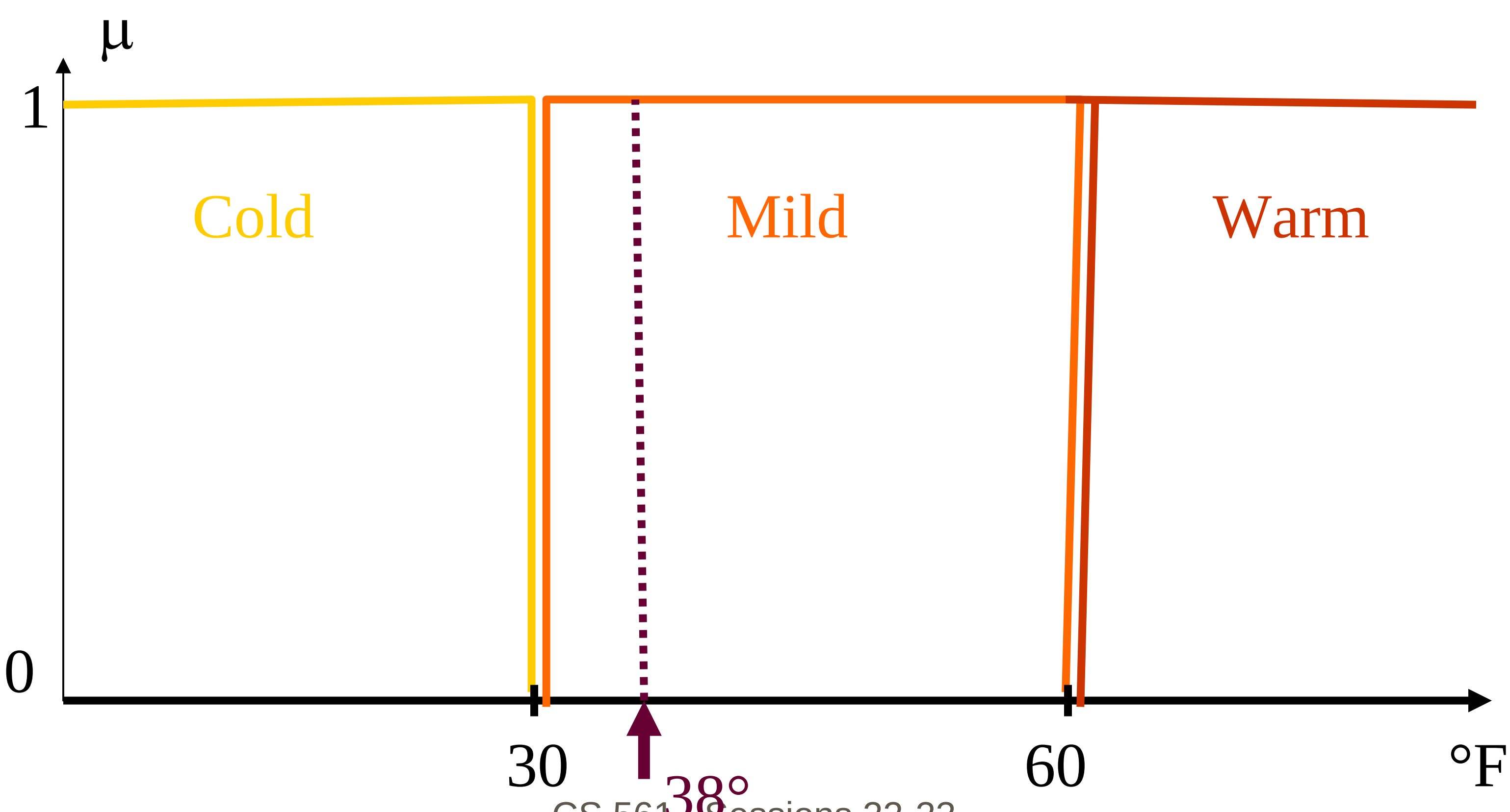
A Very Simple Example

- Fuzzification



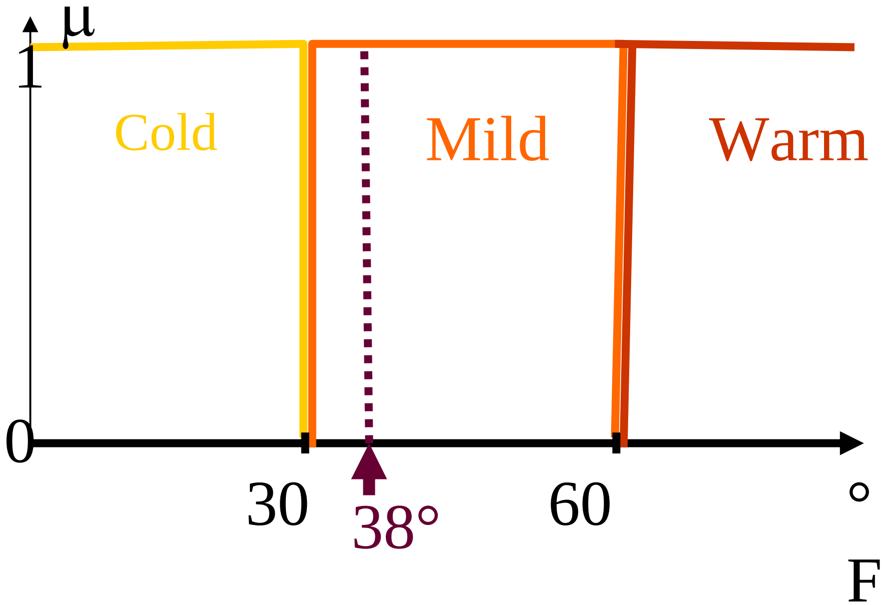
A Very Simple Example

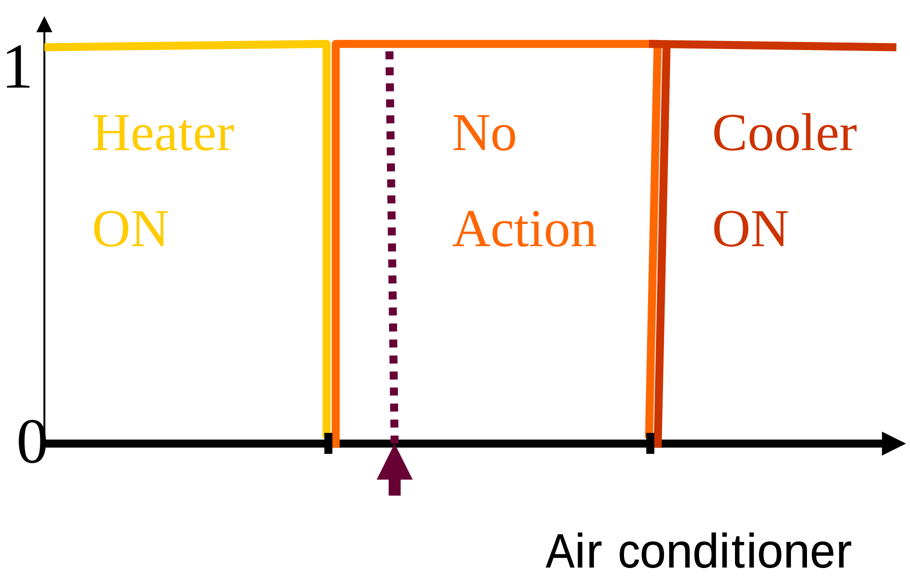
- Inferencing



A Very Simple Example

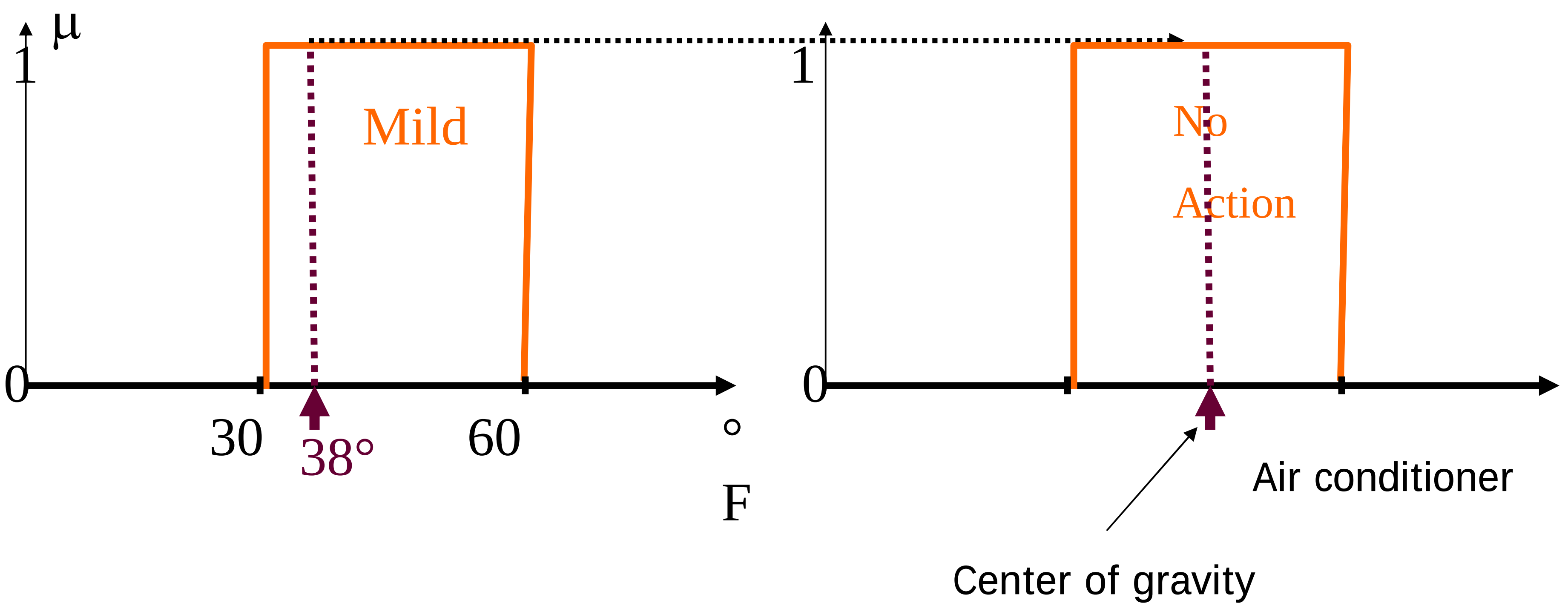
- Rule Evaluation
- If the temp. is mild, then no action



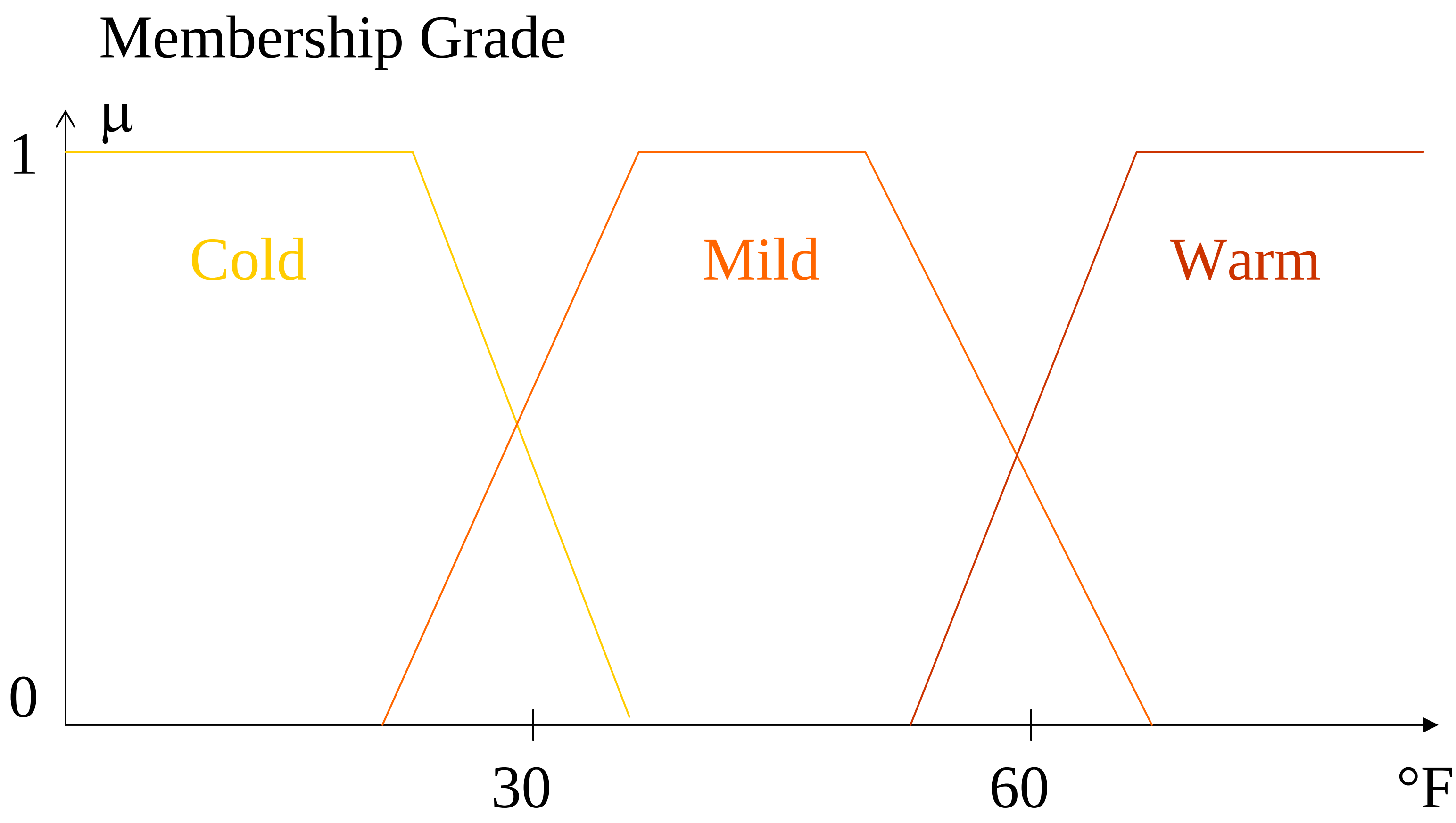


A Very Simple Example

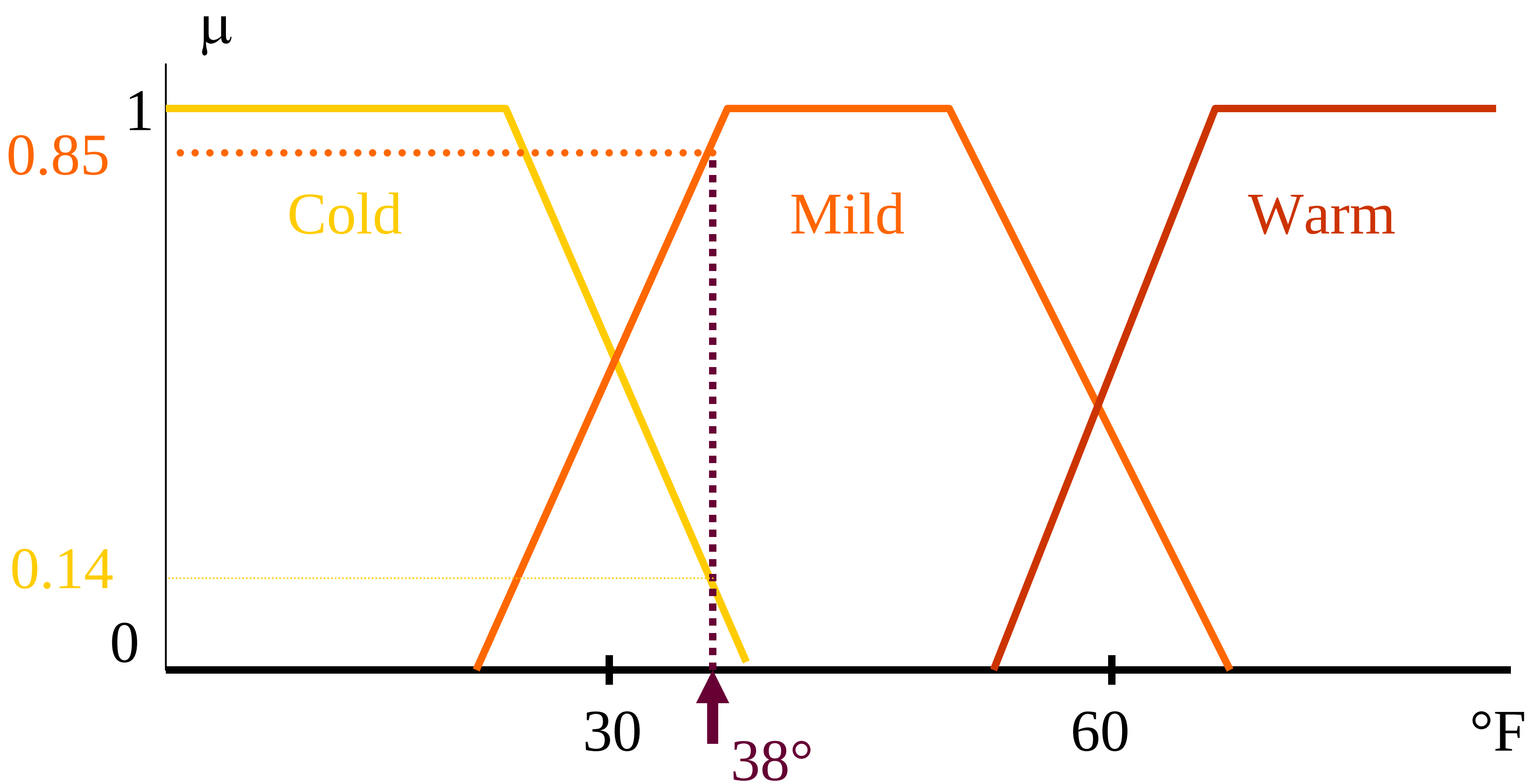
- Rule Evaluation



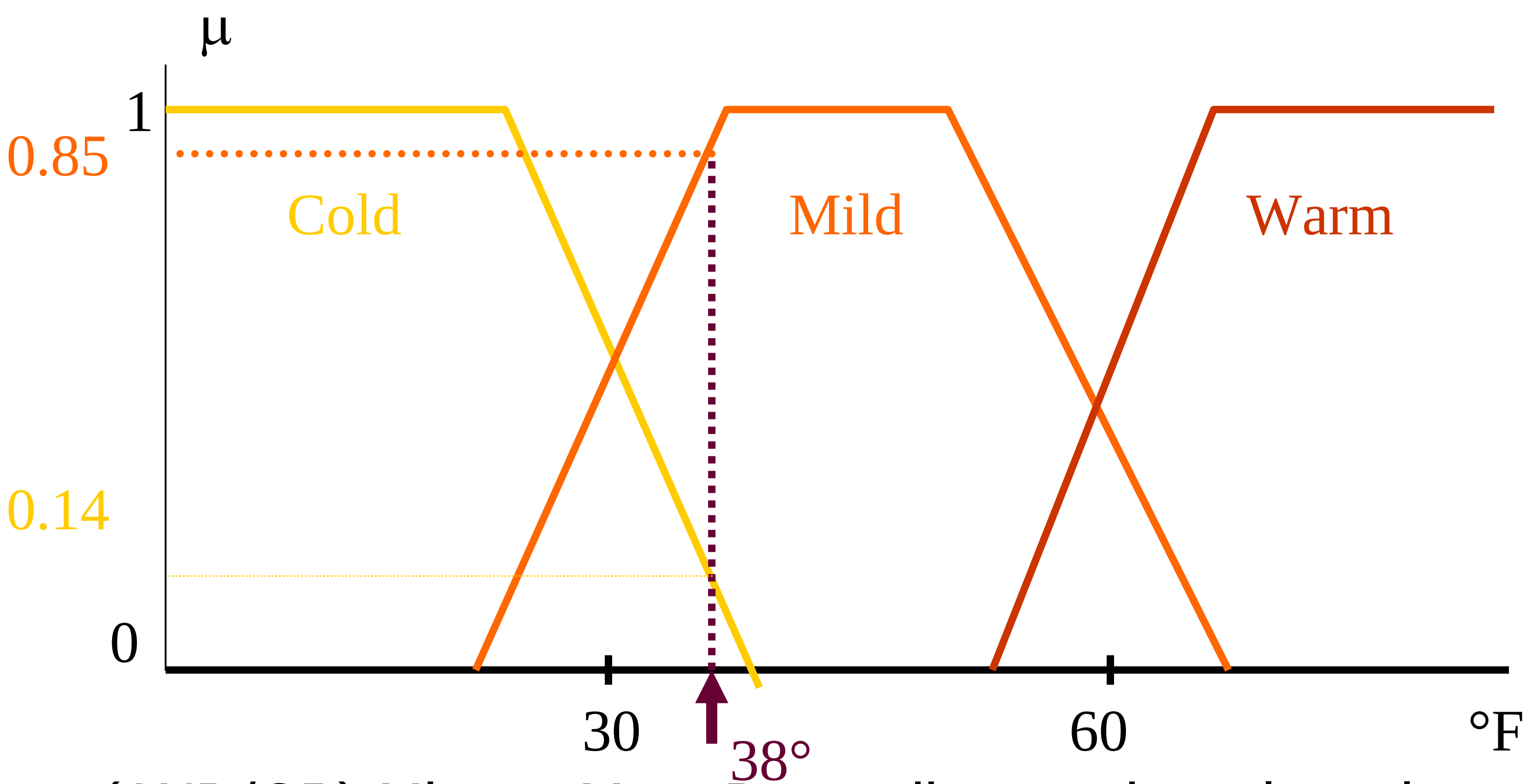
Full Example:



Fuzzification

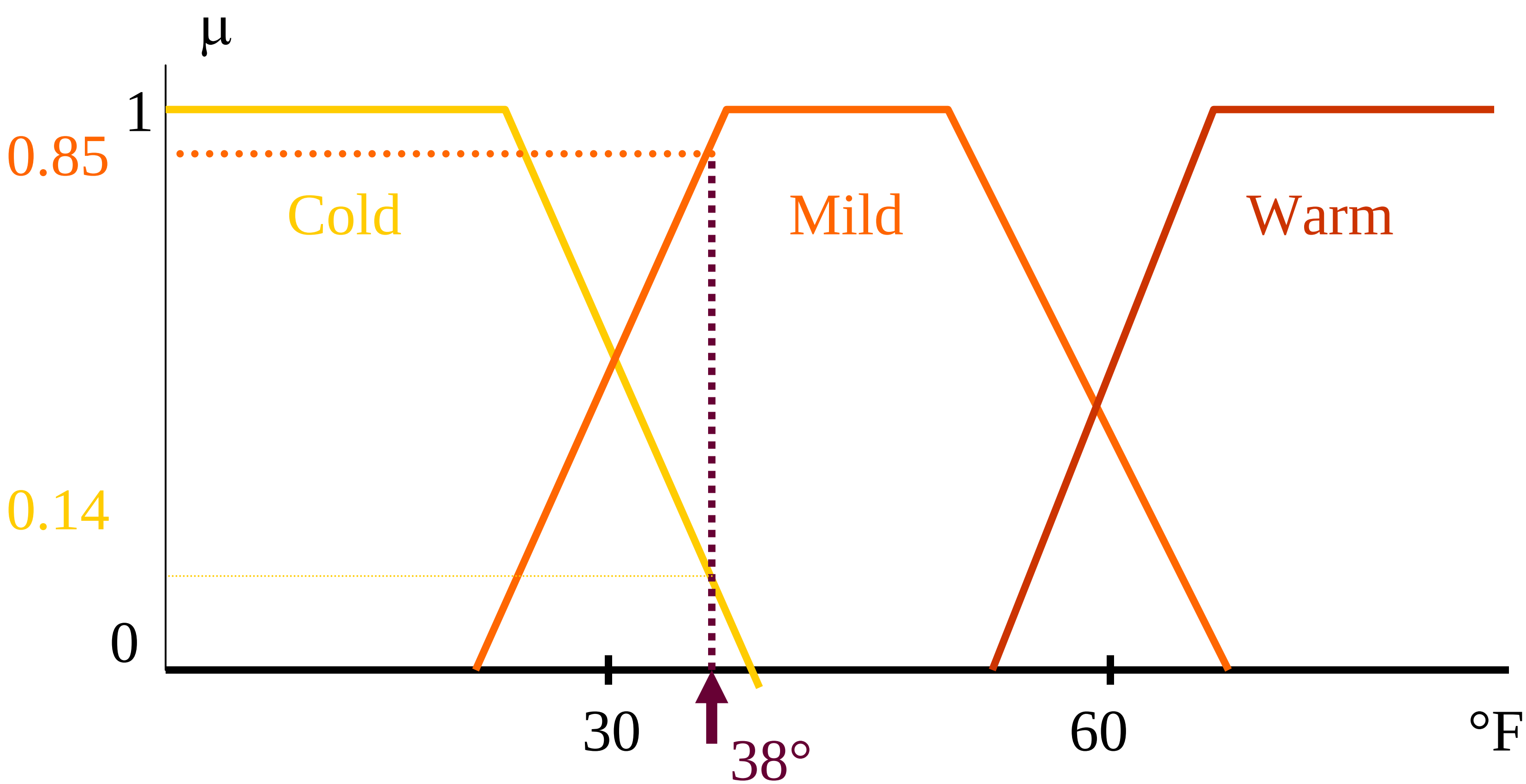


Set Operators: AND/OR



(AND/OR) Min or Max: Depending on the rule, select the value to decide the results of the rule

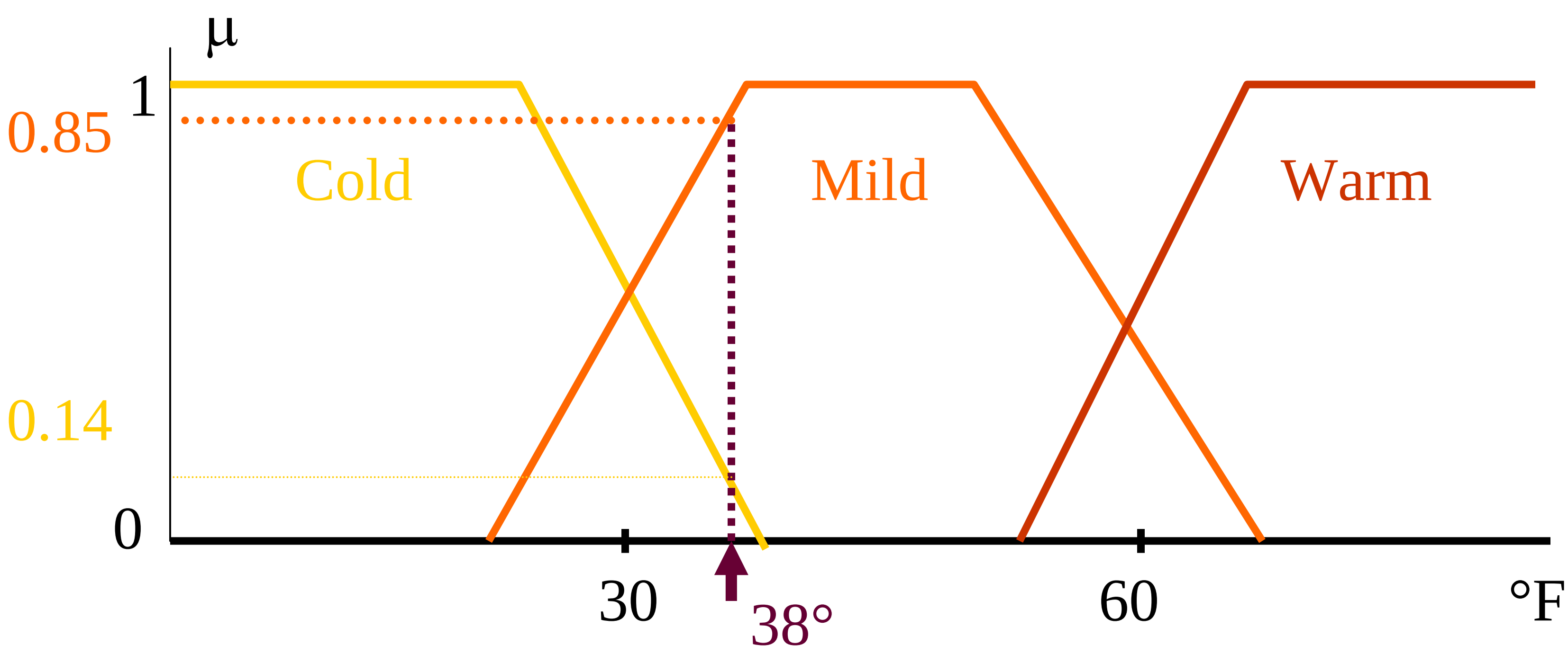
Set Operators: AND/OR



IF temp is Mild THEN No Action

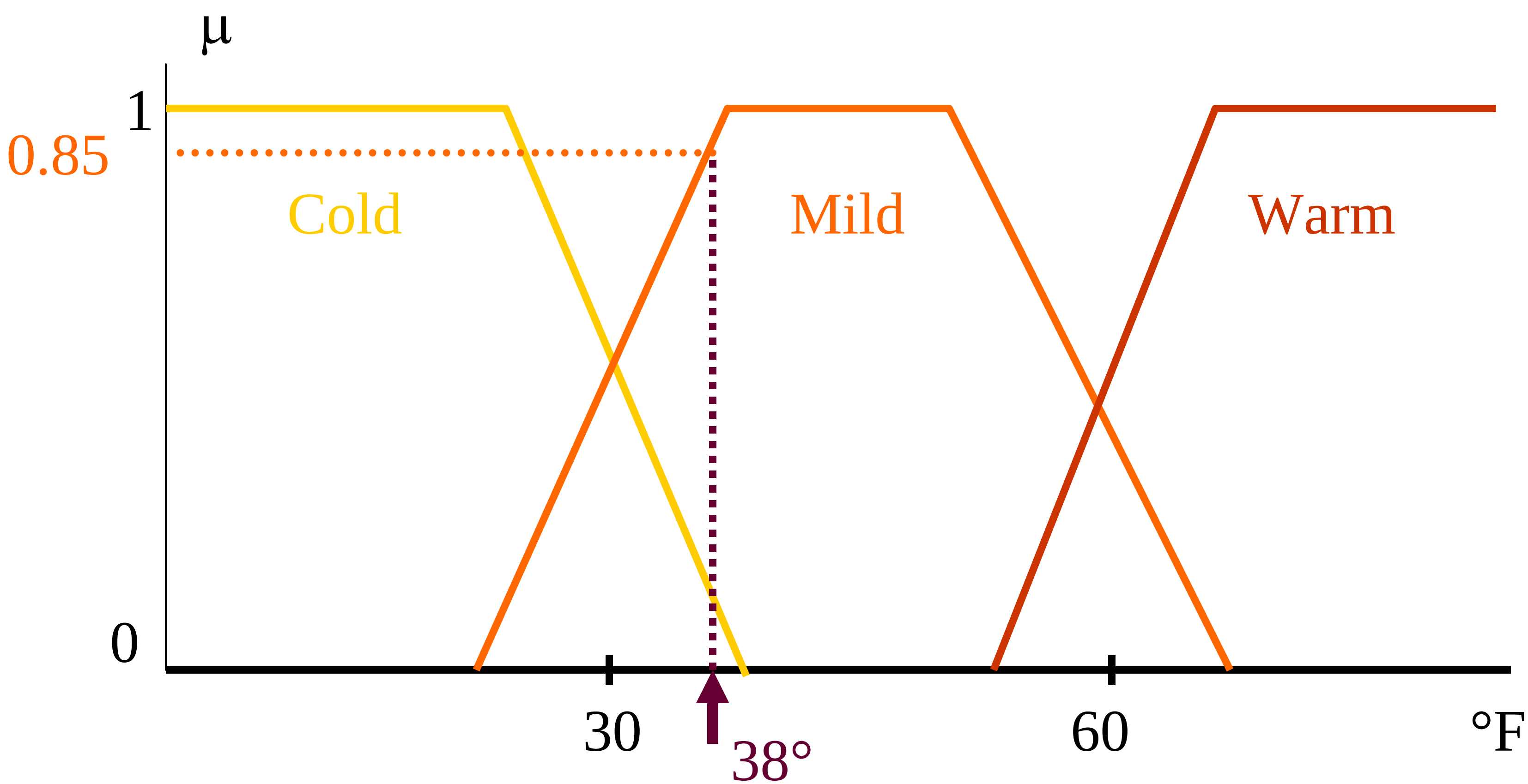
This is a single item rule. No min/max applied

Set Operators: AND/OR



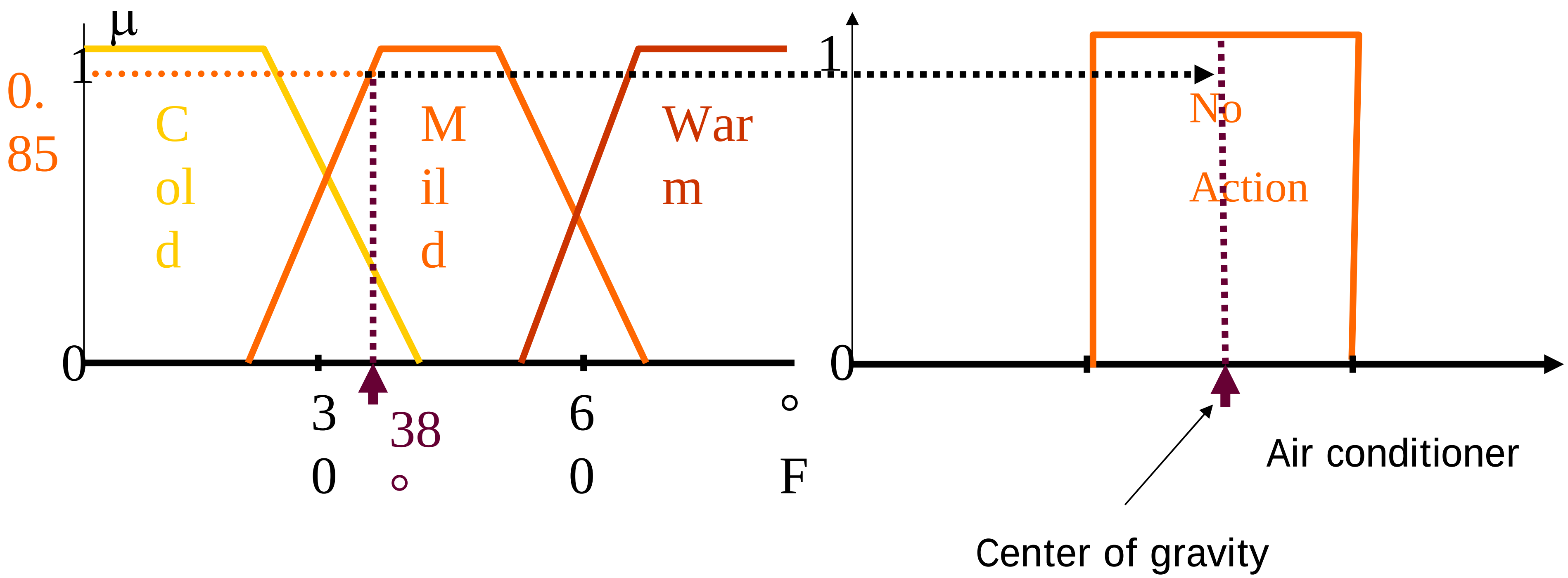
- If you have a sick kid at home and it is cold for him

Set Operators: Max value



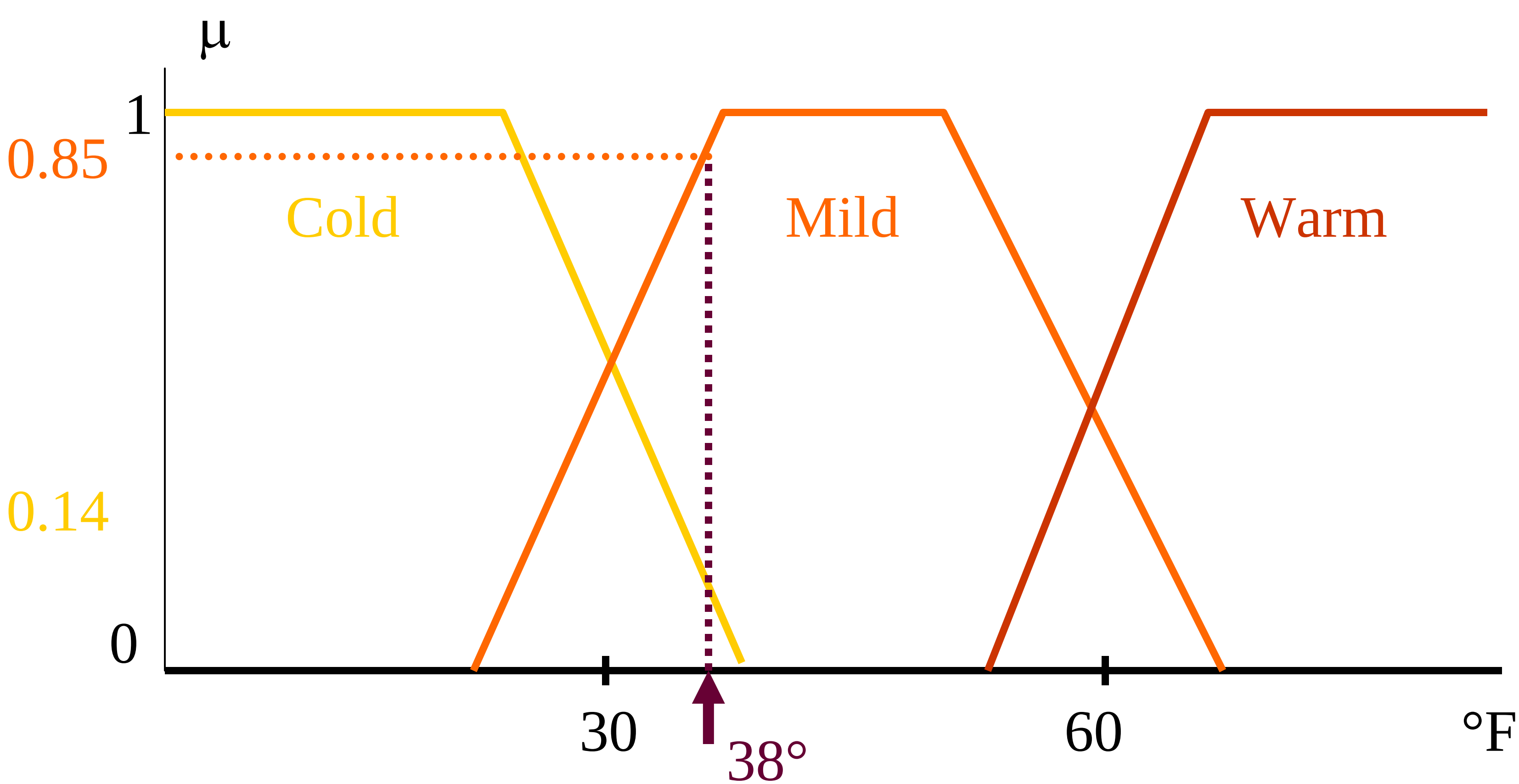
It is mild, no need for heating up

Set Operators: Max value



It is mild, no need for heating up

Question

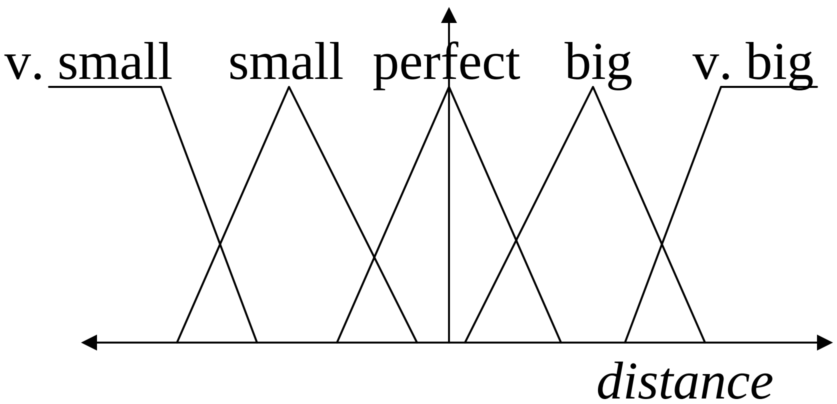


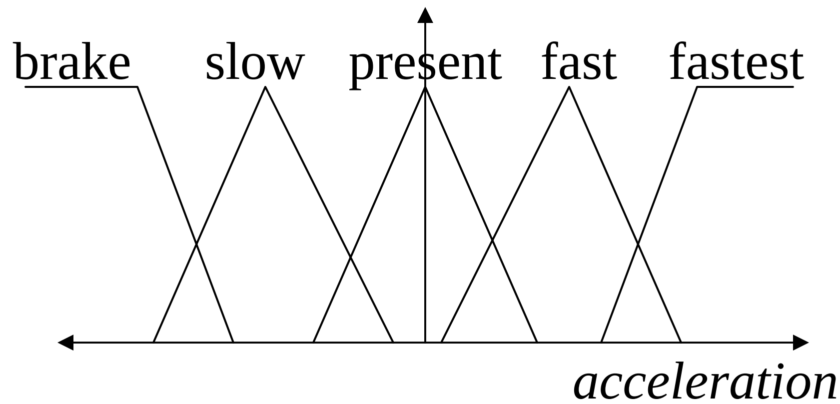
Give an example to make it depending on two inputs.

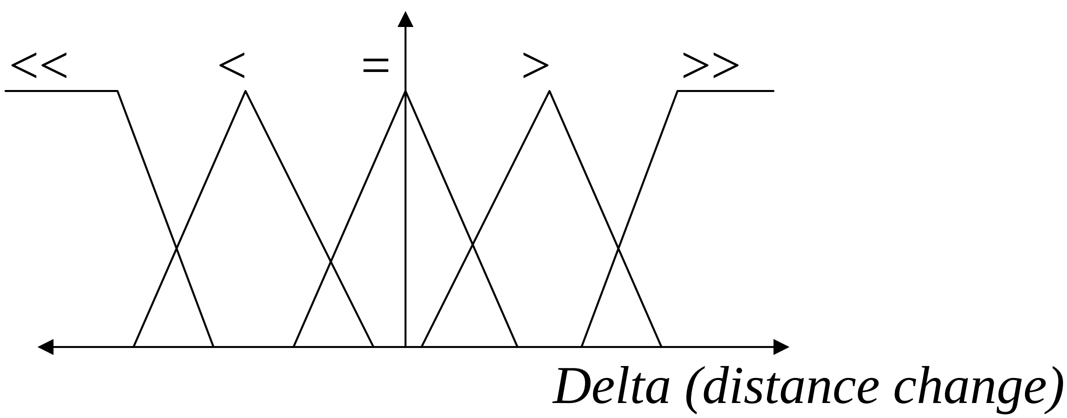
Fuzzy Rules

- Example: "If our distance to the car in front is small, and the distance is decreasing slowly, then decelerate quite hard"
  - Fuzzy variables in blue
  - Fuzzy sets in red
- *QUESTION:* Given the distance and the change in the distance, what acceleration should we select?

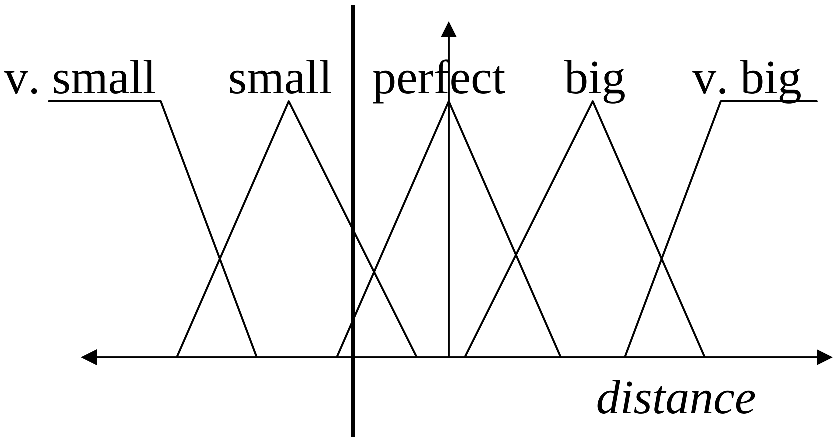
Fuzzification: Set Definitions

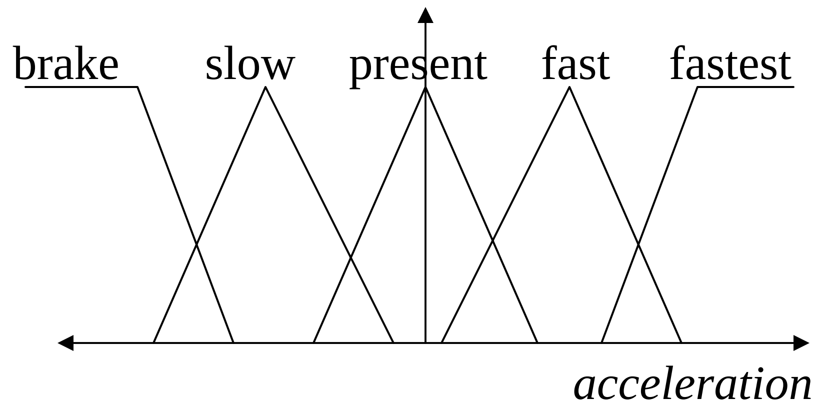






Fuzzification: Instance





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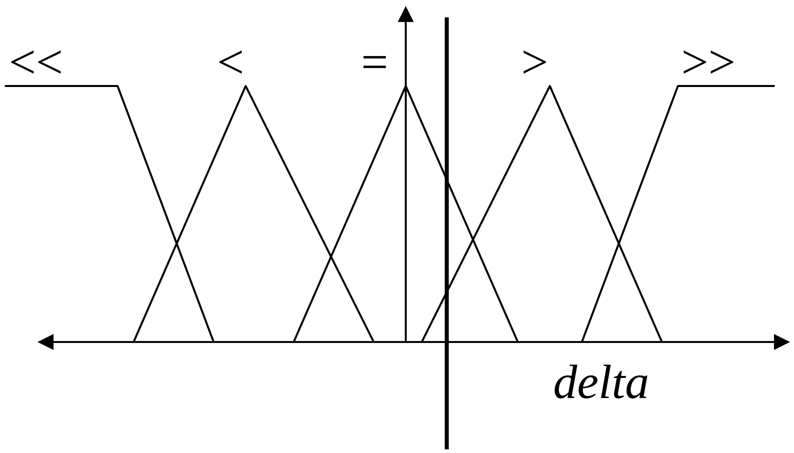
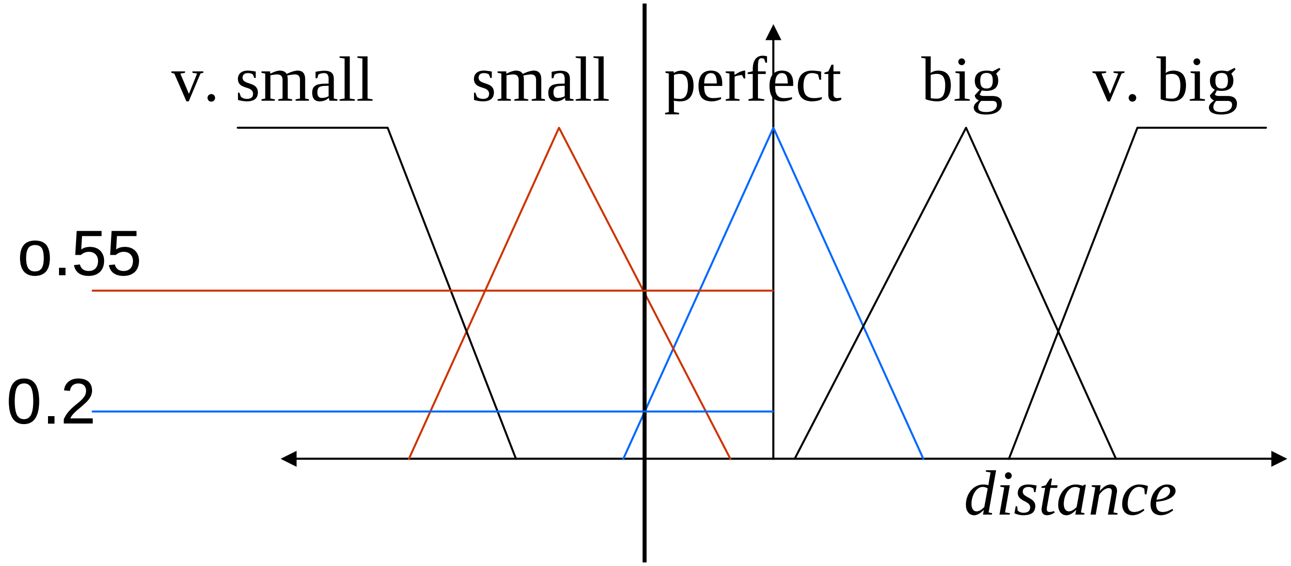


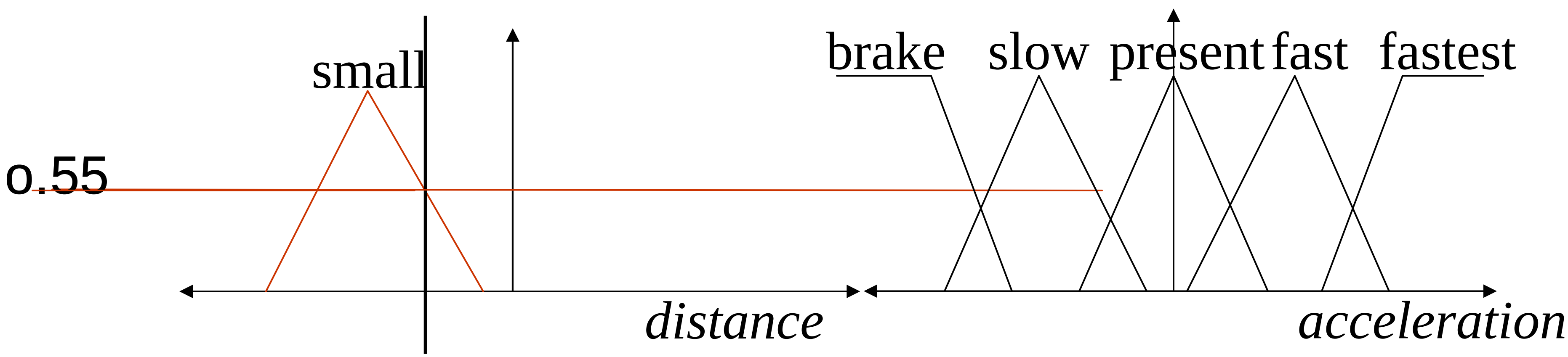
- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

Fuzzification: Instance



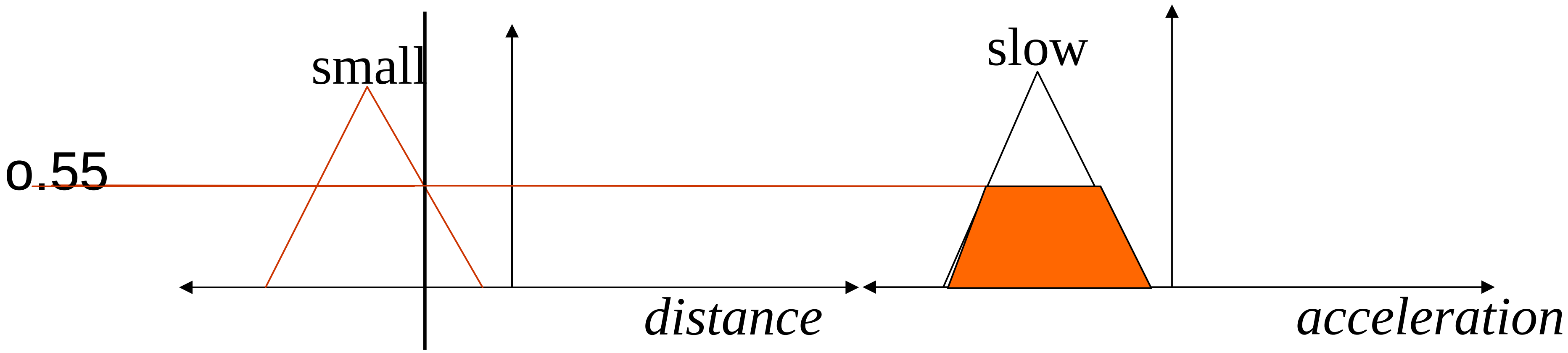
IF distance is Small THEN Slow Down

Rule Evaluation



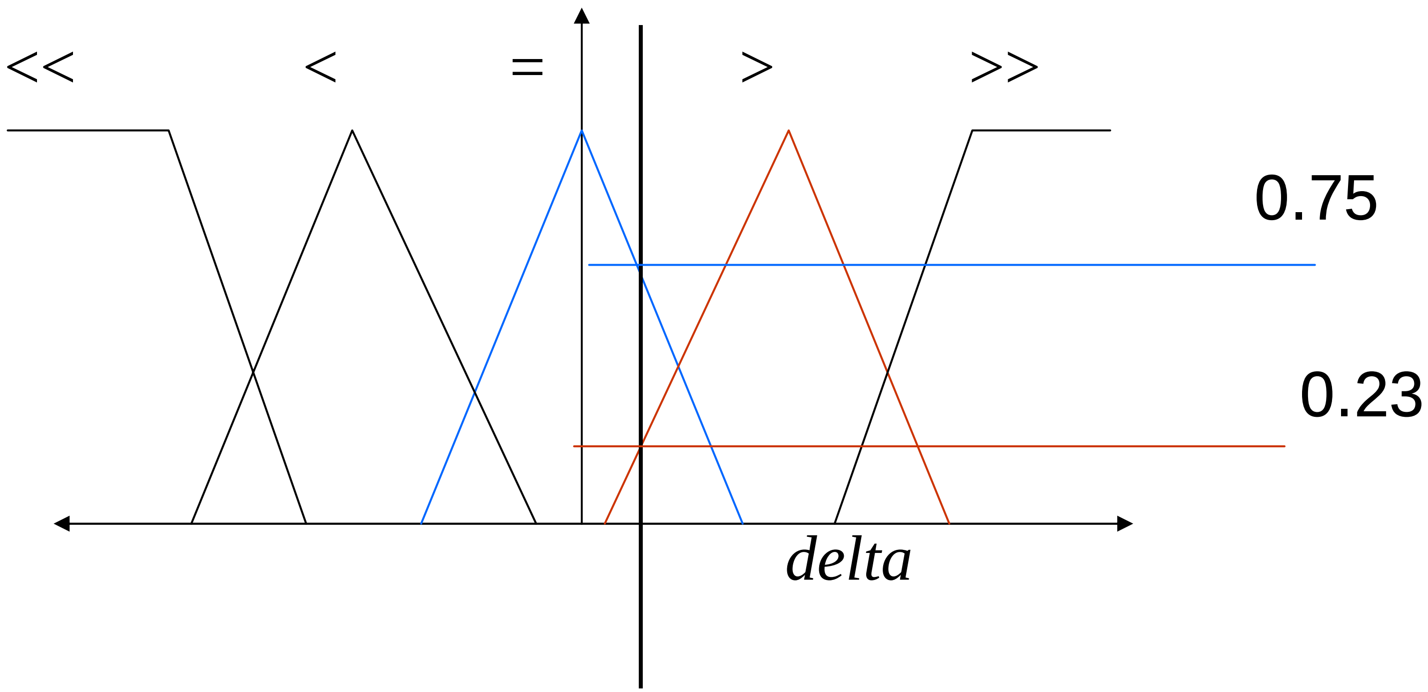
Distance is small, then you slow down.
Question: What is the weight to slow down?

Rule Evaluation



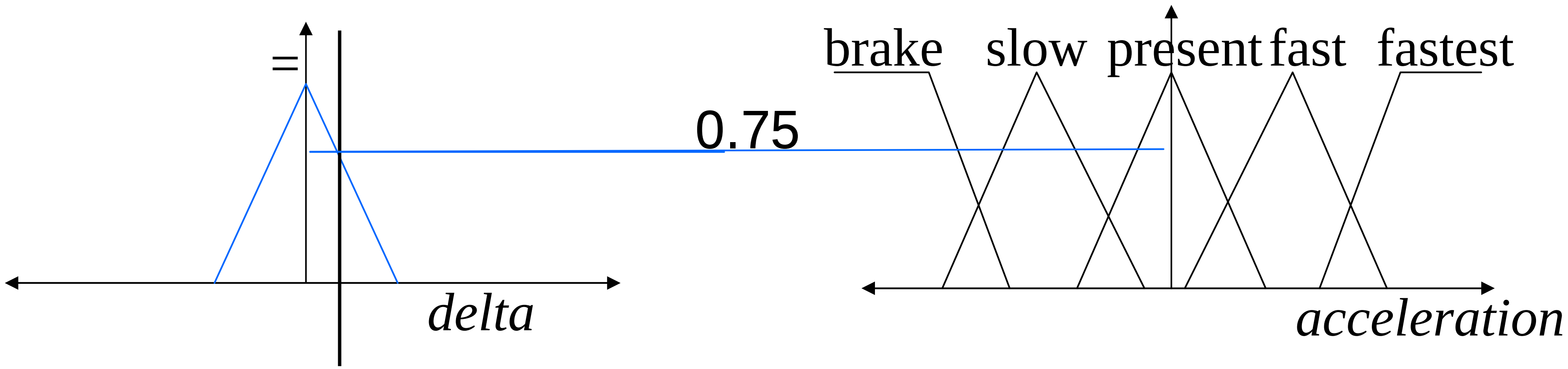
Distance is small, then you slow down.

Fuzzification: Instance



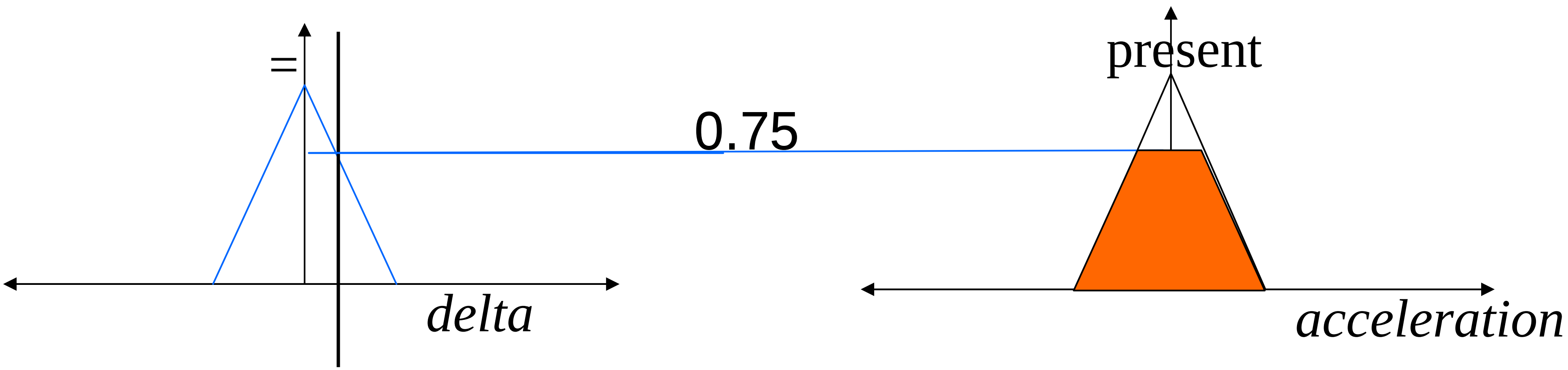
IF change in distance is = THEN Keep the speed

Rule Evaluation



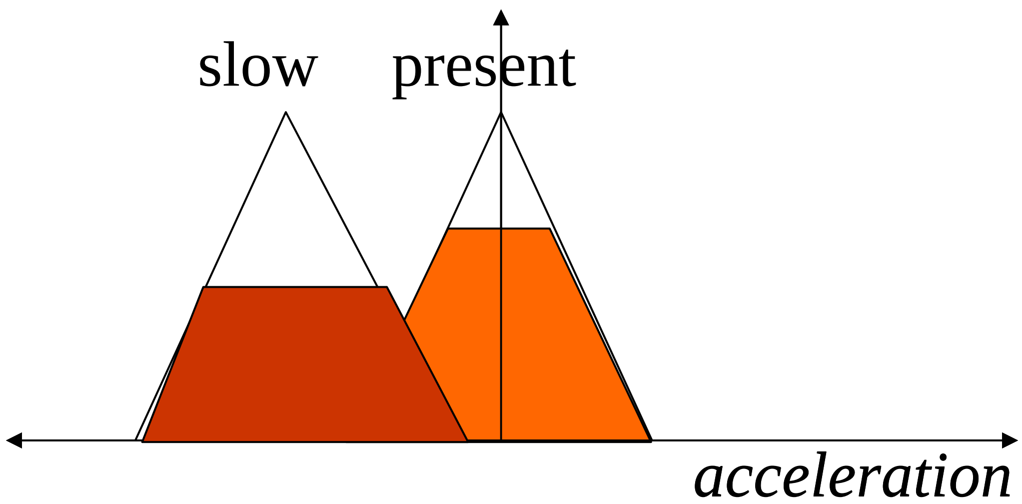
Distance is not growing, then keep present acceleration

Rule Evaluation



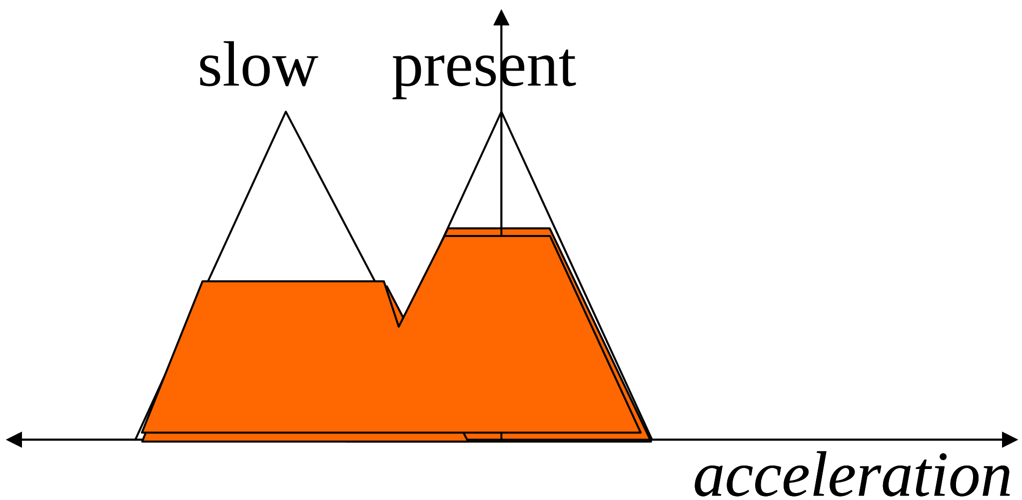
Distance is not growing, then keep present acceleration

Rule Aggregation



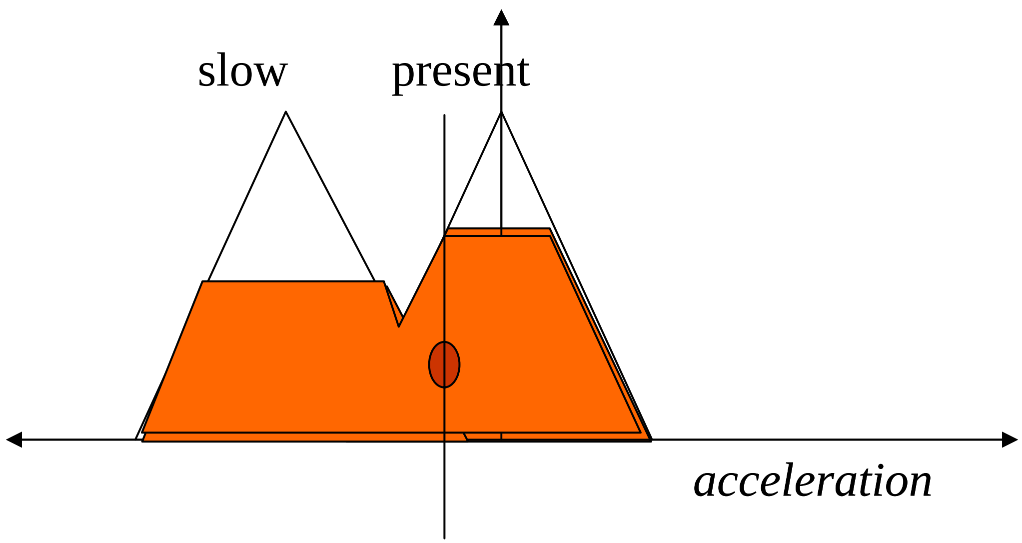
- From distance
- From delta (distance change)

Rule Aggregation



So what should we do? Current acceleration or slow down?

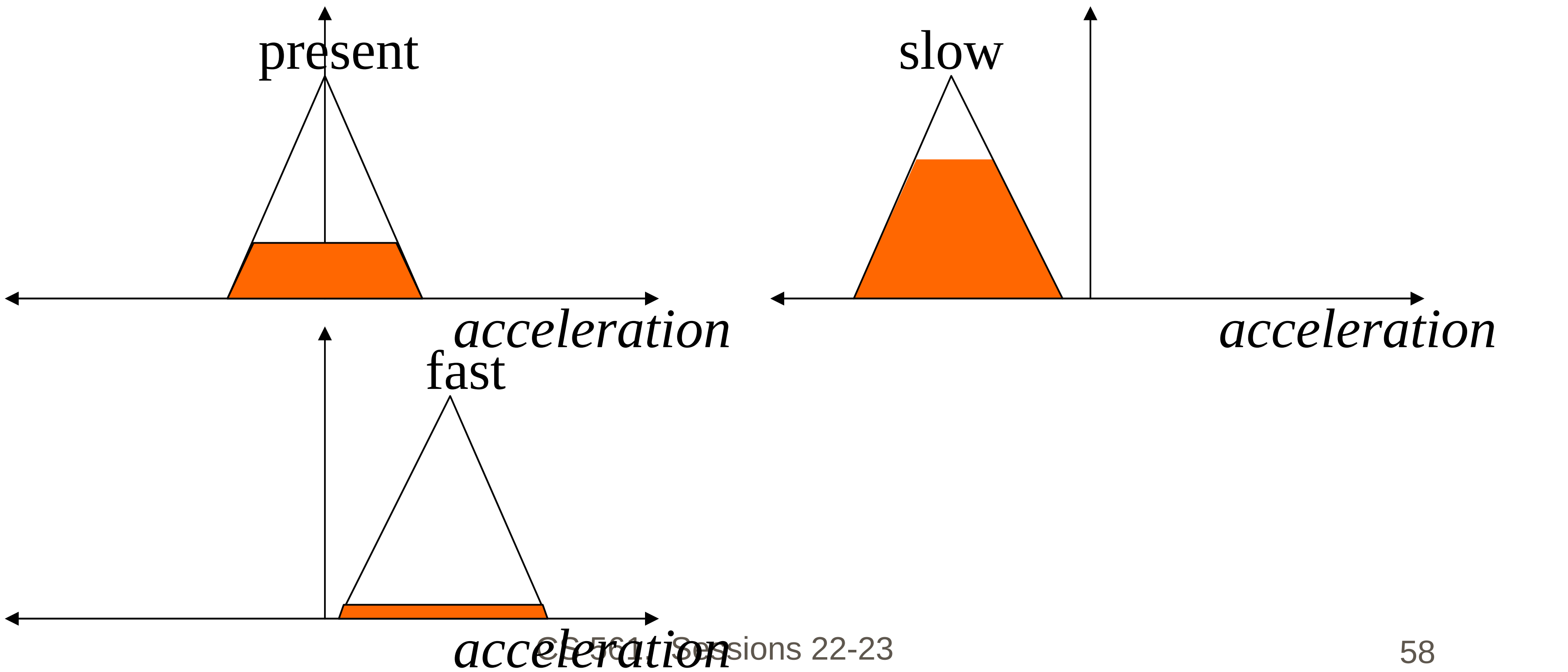
Defuzzification



So what should we do? Present acceleration or slow down?

Rule Aggregation: Another case

- Convert our belief into action
  - For each rule, clip action fuzzy set by belief in rule



Rule Aggregation: Another case

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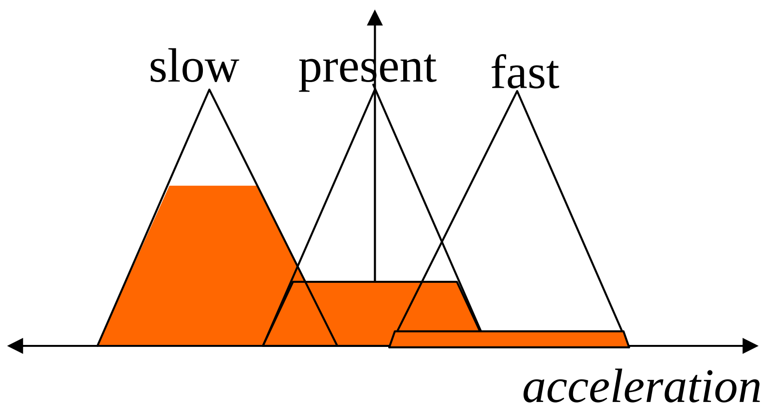


Matching for Example

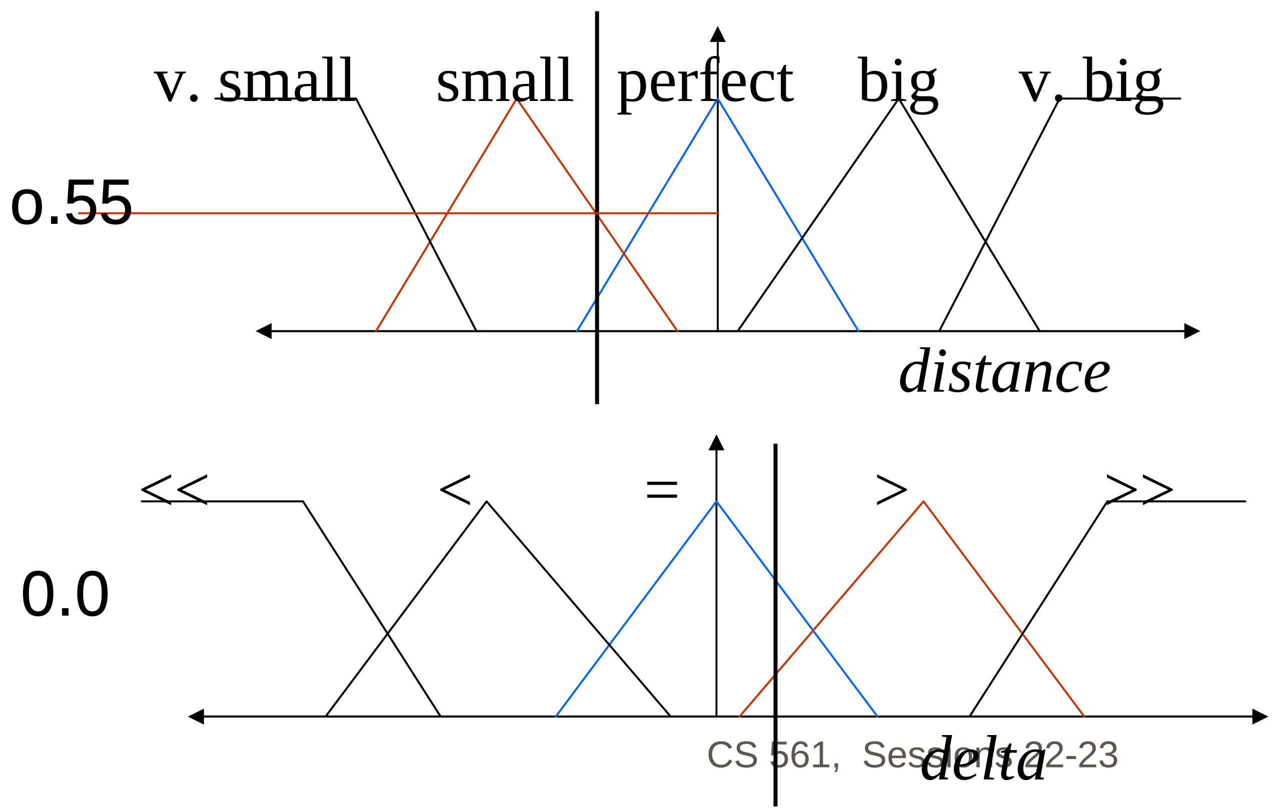
- Relevant rules are:
  - If distance is small and delta is growing, maintain speed
  - If distance is small and delta is stable, slow down
  - If distance is perfect and delta is growing, speed up
  - If distance is perfect and delta is stable, maintain speed

Matching for Example

- For first rule, distance is small has 0.75 truth, and delta is growing has 0.3 truth
  - So the truth of the **and** is 0.3
- Other rule strengths are 0.6, 0.1 and 0.1

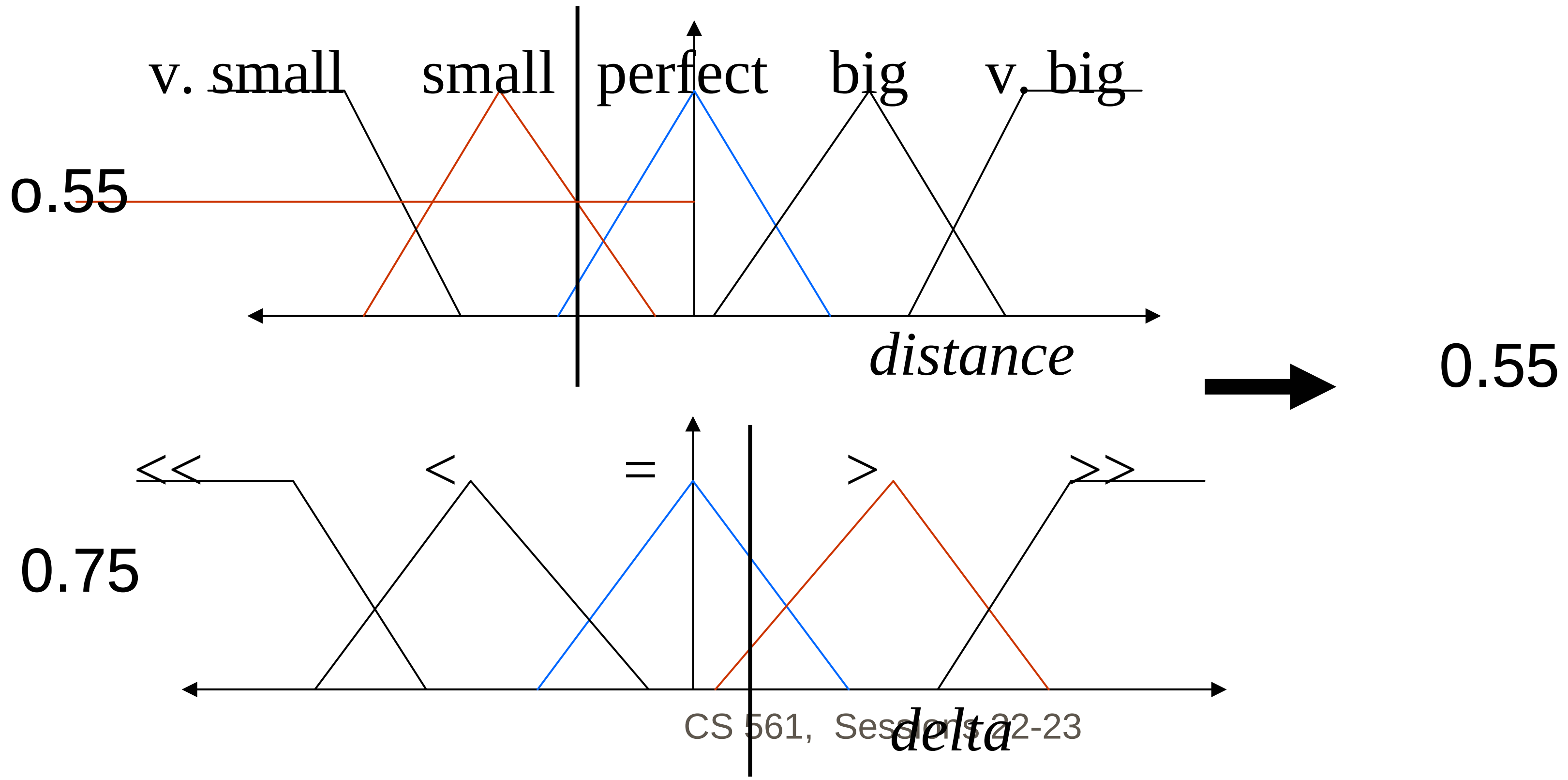
AND/OR Example

- IF Distance Small AND change in distance negative THEN high deceleration



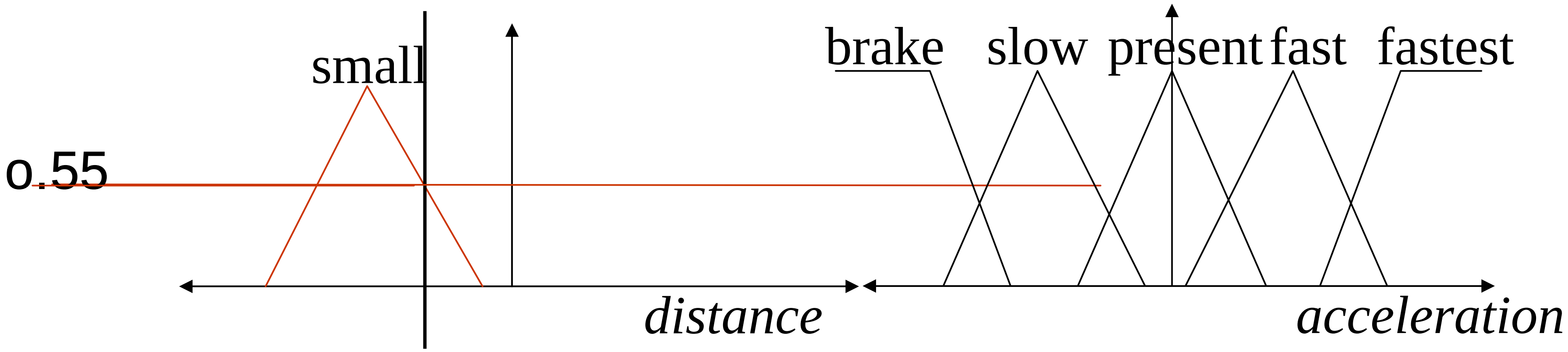
AND/OR Example

- IF Distance Small AND change in distance = THEN slow deceleration

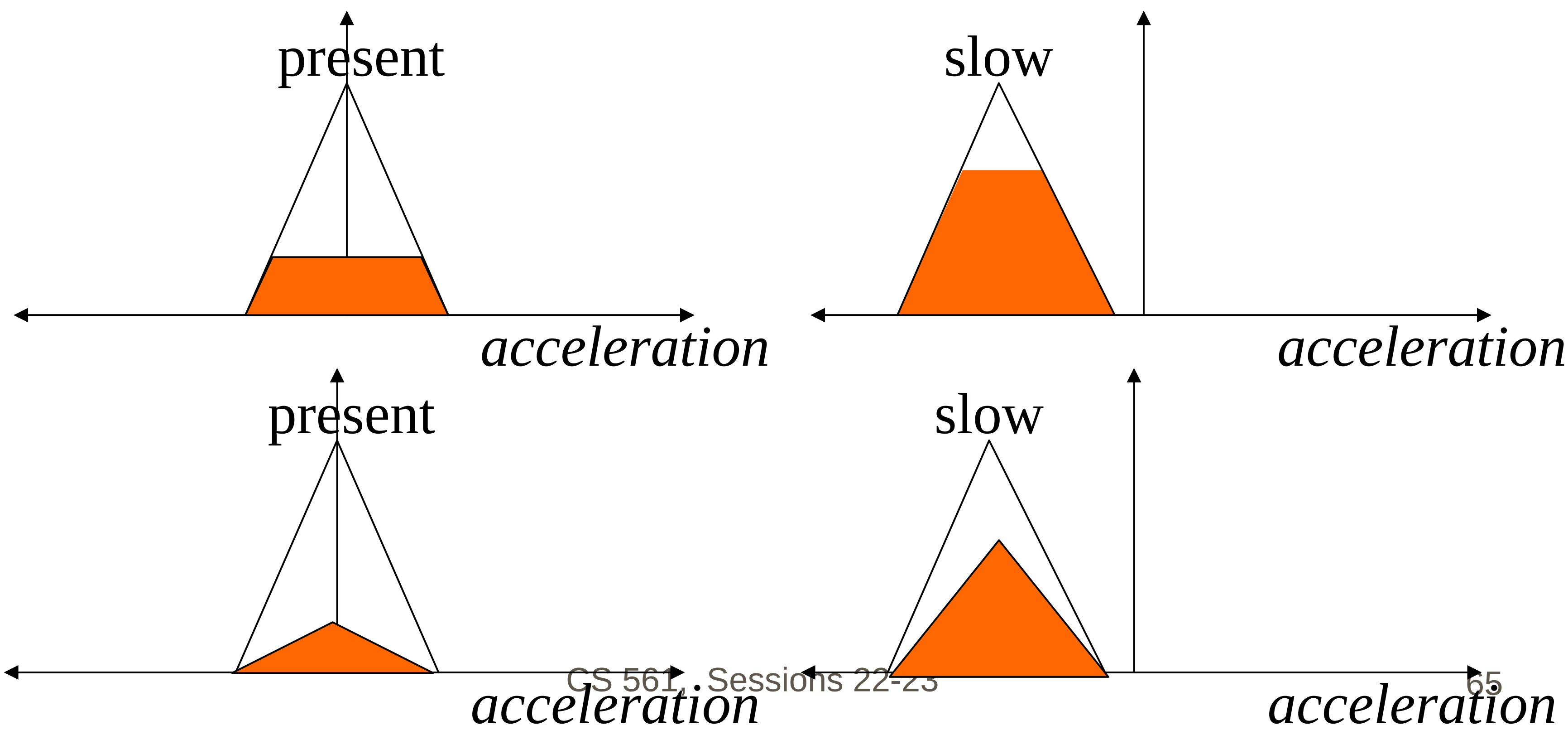


AND/OR Example

- IF Distance Small AND change in distance = THEN slow deceleration

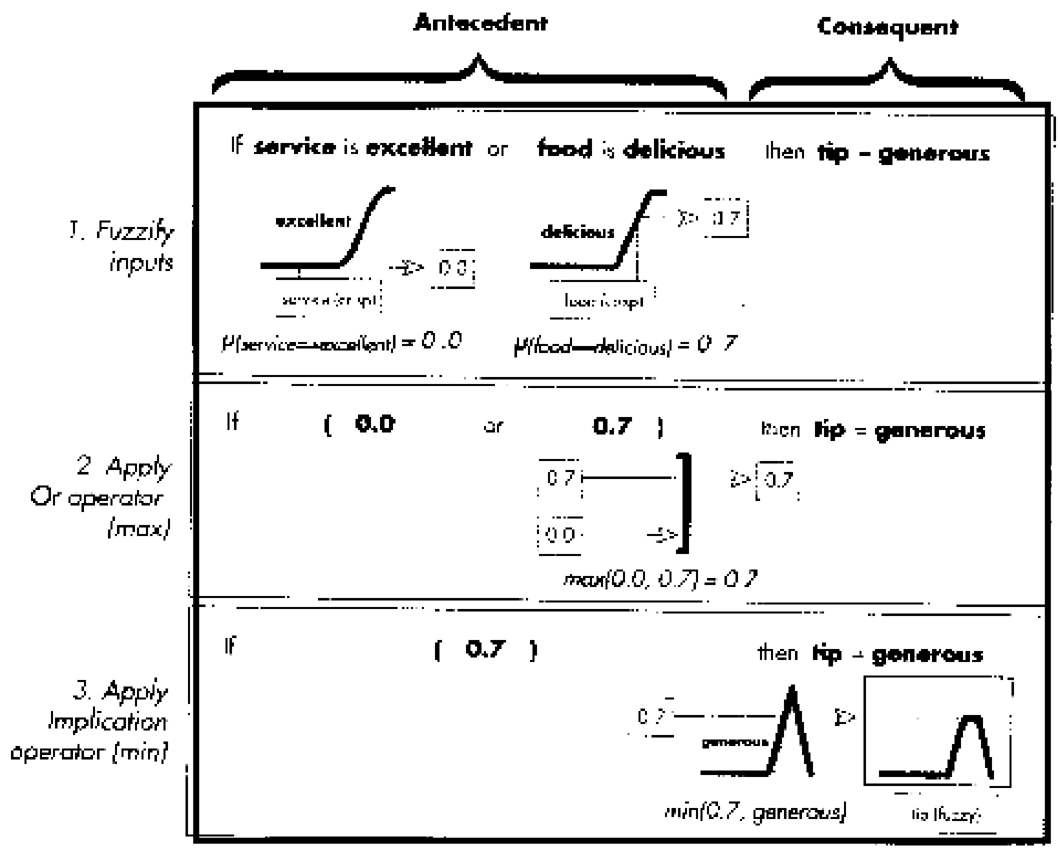


Scaling vs. Clipping



Evaluation of fuzzy rules (cont'd)

- Apply **implication function** to the rule
- Most common way is to use min to "chop-off" the consequent (prod can be used to scale the consequent)



Summary: If-Then rules

1. Fuzzify inputs:
   Determine the degree of membership for all terms in the premise.
   If there is one term then this is the degree of support for the consequence.

2. Apply fuzzy operator:
   If there are multiple parts, apply logical operators to determine the degree of support for the rule.

Summary: If-Then rules

3. Apply implication method:

Use degree of support for rule to shape output fuzzy set of the consequence.

How do we then combine several rules?

Multiple rules

- We aggregate the outputs into a single fuzzy set which combines their decisions.
- The input to aggregation is the list of truncated fuzzy sets and the output is a single fuzzy set for each variable.
- **Aggregation rules**: max, sum, etc.
- As long as it is commutative then the order of rule exec is irrelevant.

max-min rule of composition

- Given N observations E_i over X and hypothesis H_i over Y we have N rules:

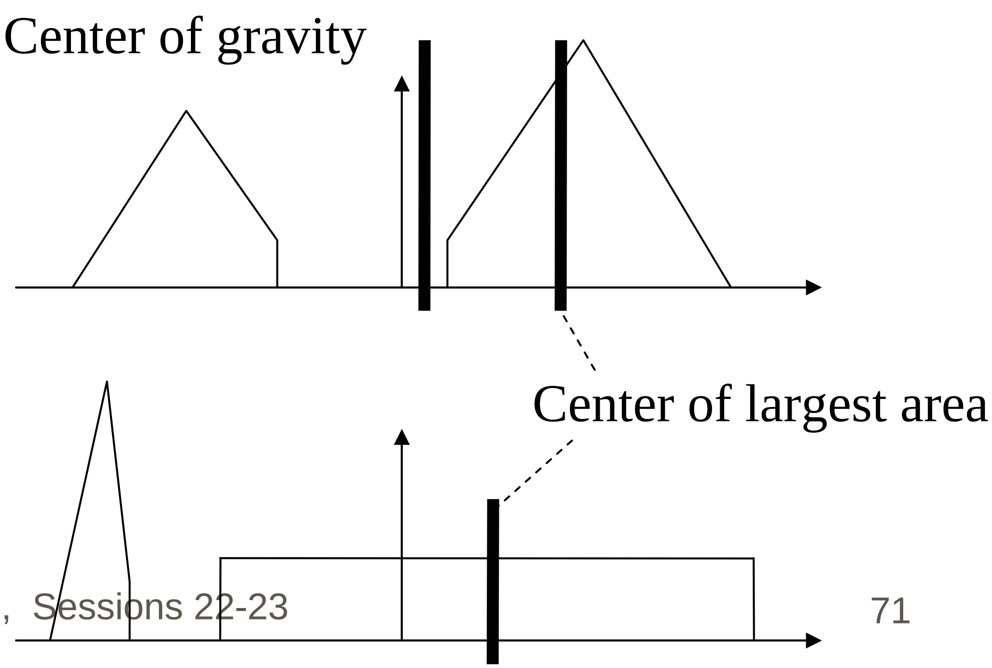
  if E_1 then H_1
  if E_2 then H_2

  if E_N then H_N
- $\mu_H = \max[\min(\mu_{E1}), \min(\mu_{E2}), \ldots \min(\mu_{EN})]$

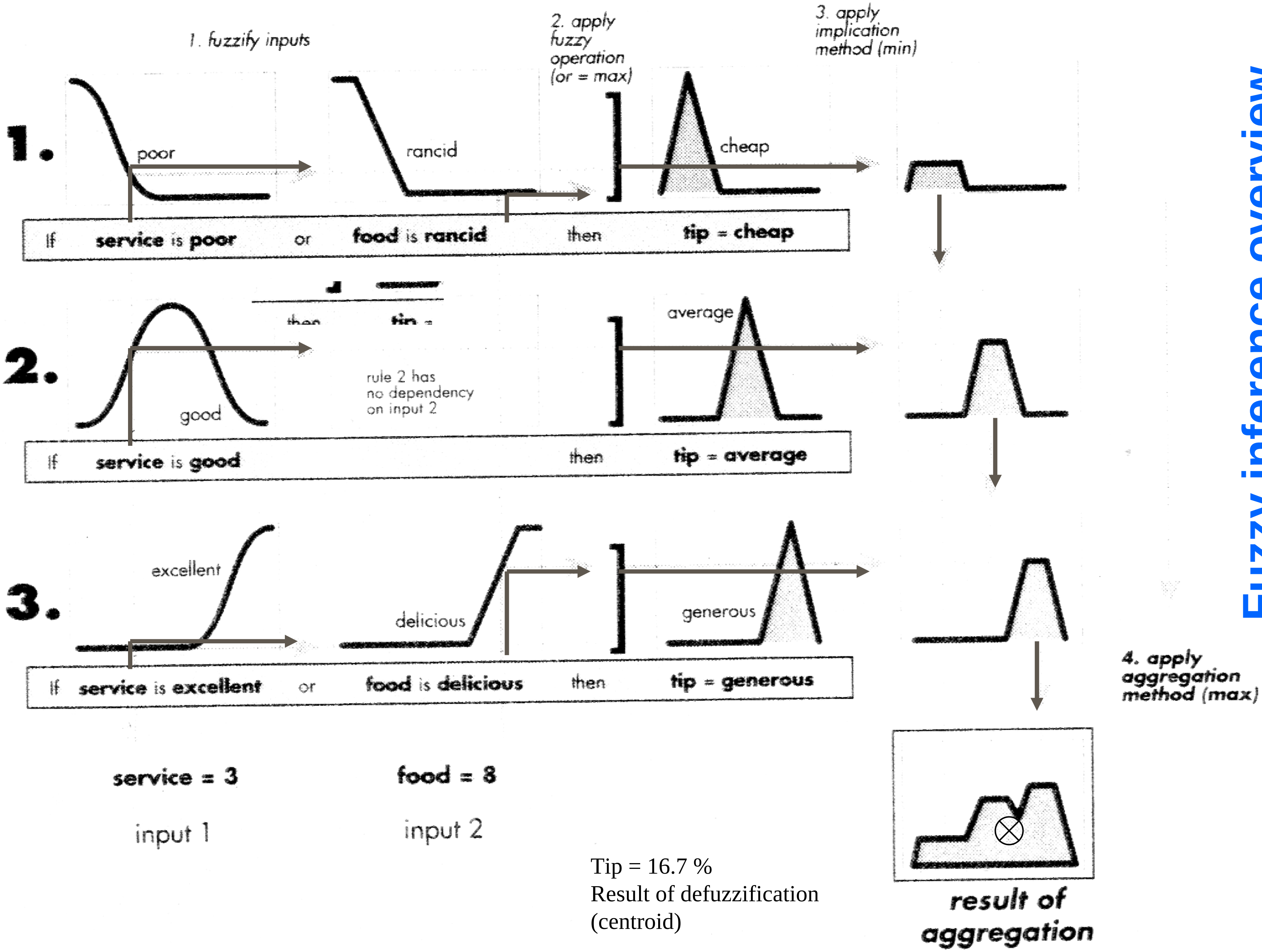
Defuzzify the output

- Take a fuzzy set and produce a single crisp number that represents the set.
- Practical when making a decision, taking an action etc.

$$I = \frac{\sum \mu_I x}{\sum \mu_I}$$



Fuzzy inference overview



Tip = 16.7 %
Result of defuzzification (centroid)

Limitations of fuzzy logic

- How to determine the membership functions? Usually requires fine-tuning of parameters

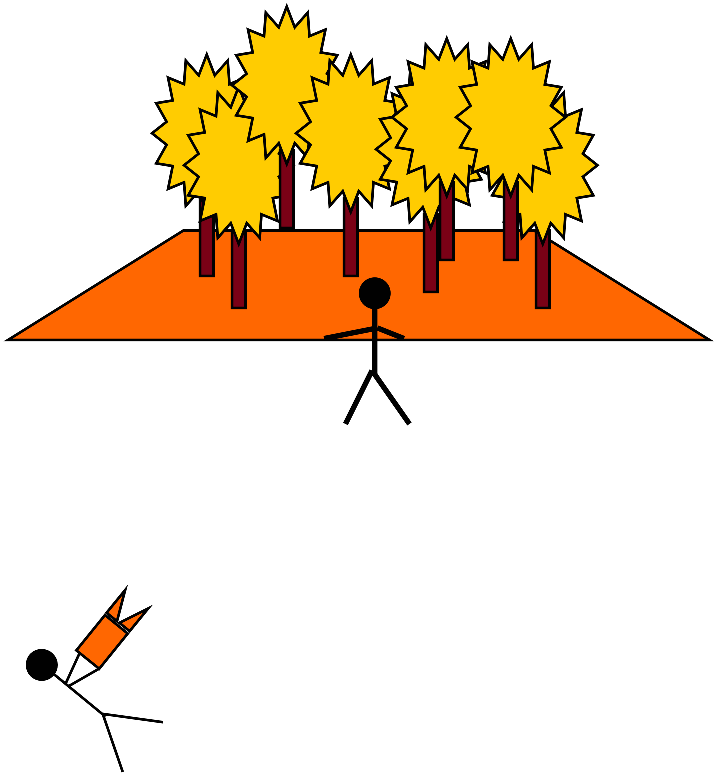
- Defuzzification can produce undesired results

Fuzzy tools and shells

- Matlab's Fuzzy Toolbox
- FuzzyClips
- Etc.

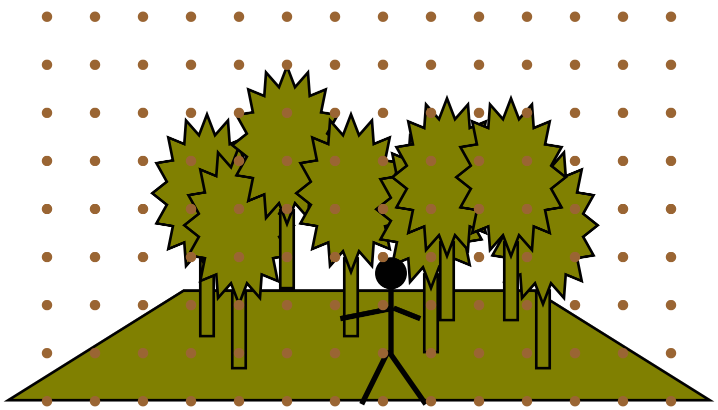
Camcorder Example

- Stabilizer operates by attempting to identify the subject versus the background. Using this, we can determine whether it is the subject and/or background that is moving, or if it is the holder of the camcorder that is moving.



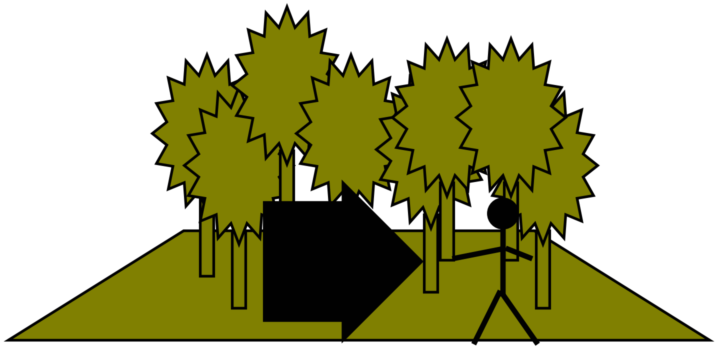
Camcorder Example

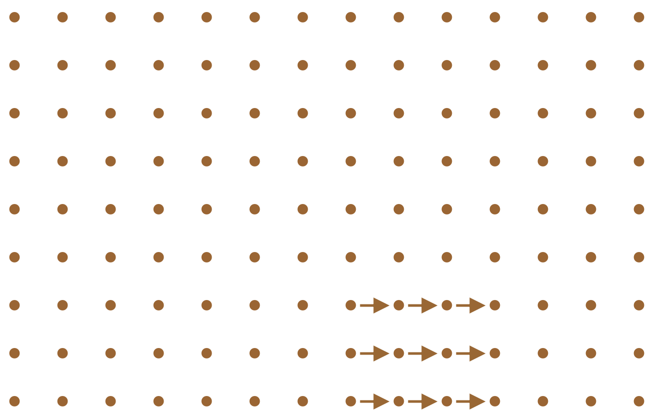
- One method is to use a set of input points in a grid and poll those points twice per second.
- Between pollings, the camcorder deduces which direction the objects have shifted....



Camcorder Example

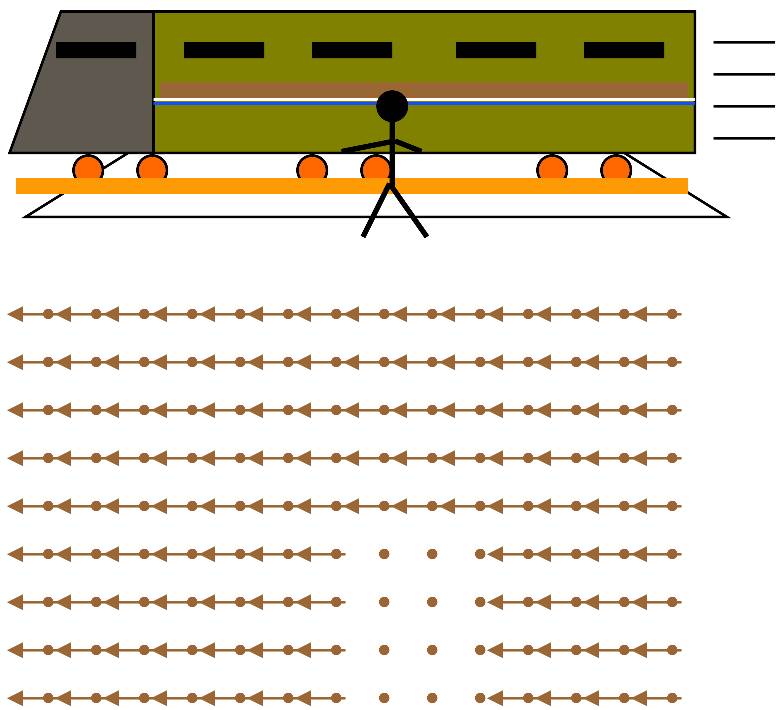
- If the subject moves, then the camcorder detects a shift among points in a localized region. If this region is somewhere in the lower center of the shot, then the chances that it is a subject-move is even greater.





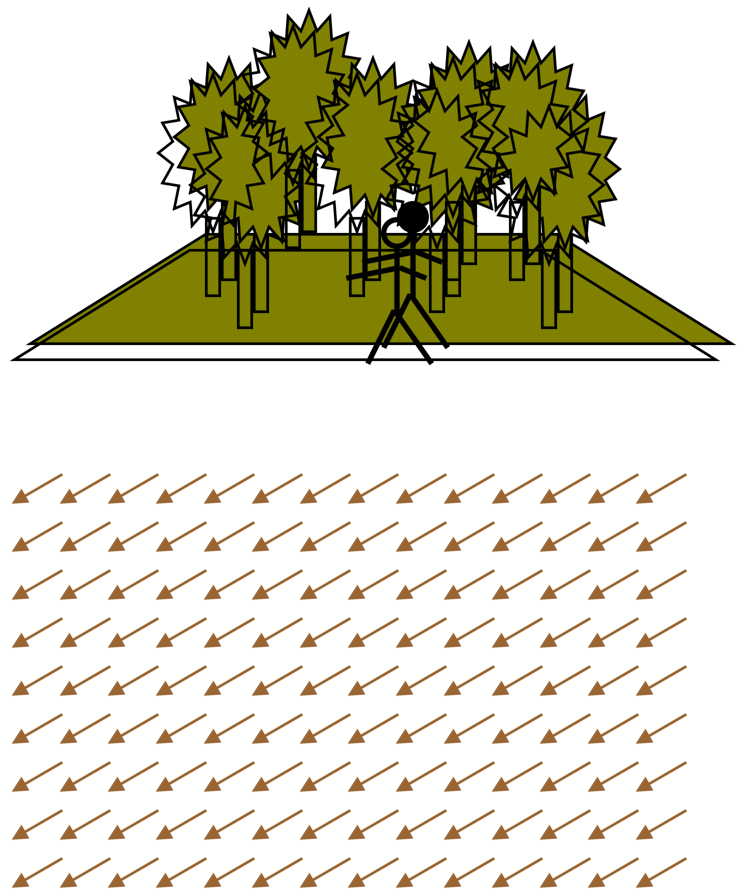
Camcorder Example

- However, if it appears that a significant amount of the screen is shifting yet a localized region is standing still, then the camcorder can deduce that the background is moving while the subject is not.



Camcorder Example

- If it appears that the entire picture has shifted and that there is no distinction of subject or background, the camcorder can identify this and deduce that the camera-holder's hand has shifted. The camcorder can then compensate for the shift.



Camcorder Example

- The fuzzy logic would work as follows:
  - Fuzzification: The fuzzy set could be: red, orange, yellow, …, purple, black, and white. Each pixel is identified as having a degree of each of these colors based on the levels of red, green, and blue detected.
  - Inference: First layer of rules deduce where the shifts occur among single pixels. Second layer of rules clump together like shifts into shifted regions.
  - Composition: Based on the collected evidence, deduce overall shift of camcorder (slight up-down, slight left-right, ….)
  - Defuzzification: Translate the overall shift of camcorder into compensatory action (slight up-down: shift picture up 1 pixel…)