Processing Geometry: Graphics as a Mathematical Discipline

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Computer Graphics (I)

Creation of visual representations of real or imagined worlds with predictive power

» modeling
» rendering
» animation

» computational geometry

Computer Graphics (II)

Various targets:

- CAD, design of surfaces
  commerce, art …
- AV productions:
  entertainment, education …
- Simulators:
  surgery, virtual reality …

Recent Developments

After the early years…

- Rasterization, z-buffer, basic shading, etc.
- Basic libraries: OpenGL, DirectX, Java 3D

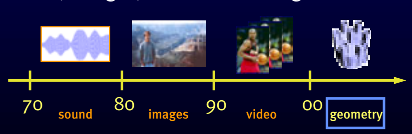
We need more accuracy

- Photorealistic radiosity: simulation of light interaction
- Scientific and medical visualization: 3D viz of rich data
- Digital geometry: handling 3D geometry with ease
- Realtime animation: realistic simulation with haptics
- Etc (geophysics, spatial exploration, biology,…)

History of Multimedia

Success of Digital Signal Processing (DSP)

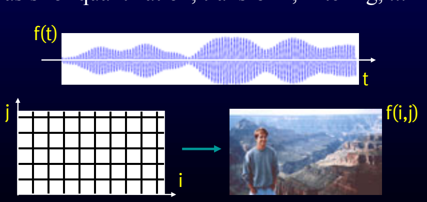
- Technology followed scientific evolution
  » is your cell phone off?
- Sounds, images, videos are all signals…



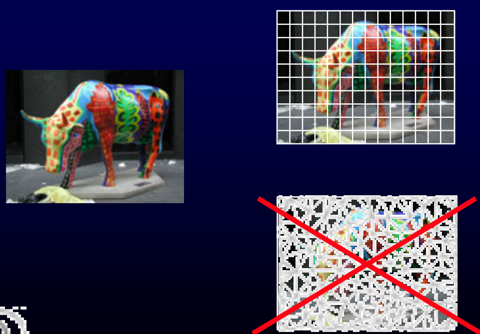
Sound, Images, Video

Signals as functions

- regular sampling
- basis for quantization, transform, filtering, ...



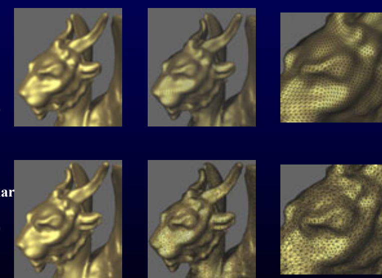
Example: Image



3D Surface (I)

Sometimes smooth, regular (6 neighbors)

Often irregular (arbitrary # of neighbors)



3D Surface (II)

Alas, DSP foundations and methods do not apply

- Irregular sampling (bye-bye, Fourier analysis)
- Curved geometry (differential geometry, non-linear)
- Topology (new parameter)

Almost all the usual DSP tools need to be re-invented!

Introducing *Digital Geometry*

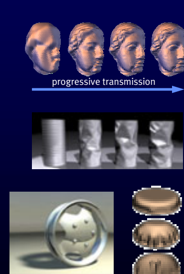
Applications

- e-catalogs
- mass customization
- electronic games
- medicine & biology
- art history & archeology
- reverse engineering

Geometry Processing (DGP)

Lots of things to do…

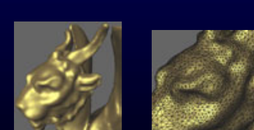
- creation, acquisition
- storage, transmission
- authentication
- editing, animation
- simulation
- manufacture

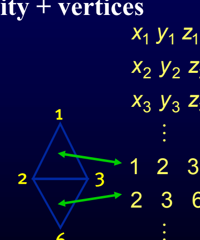


Example: DGP Compression (I)

Triangle mesh = connectivity + vertices

- dense
- irregular connectivity

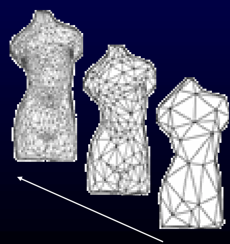




Example: DGP Compression (II)

Connectivity encoding

- less than 3.24b/v (theoretical limit – see graph theory)
  - single-rate:
    - Touma et al. 98
    - Alliez at al. 2001
    - Lee et al. 2002
  - progressive:
    - MPEG4: Taubin et al. 98
    - CPM: Pajarola et al. 99
    - Cohen-Or et al. 99
    - Alliez et al. 2001



Example: DGP Compression (III)

Geometry encoding

- methods for “large” geometry
- progressivity critical

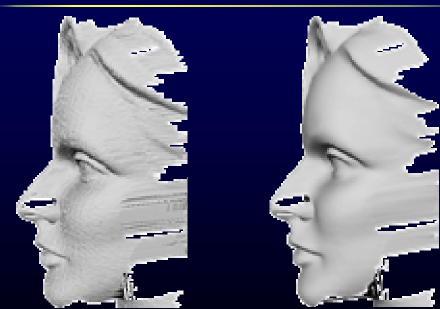
Example: Smoothing Arbitrary Meshes

Context:

- Meshes often come from 3D scans
- Noisy
- Non uniform triangles
- No parameterization, just positions in space

Smoothing : removal of rough detail features while preserving the global shape

Results on 3D Scanned Data



Initial mesh   After one fairing

More Results

Initial mesh

More Results

After upsampling and one fairing

How to Denoise Shape?

Curvature flow on the geometry

$\dot{\mathbf{x}} = -\kappa\,\mathbf{n}$ **(Laplace-Beltrami)**

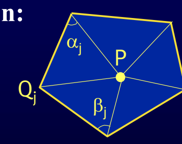
Problem: how can we find $\kappa\,\mathbf{n}$ reliably?

- Needs to be accurate whatever the # of neighbors
- Perfectly 0 if perfectly flat

Discrete Curvature

Differential geometry definition:

$$\kappa\,\mathbf{n} = \frac{\nabla_P A}{2A}$$

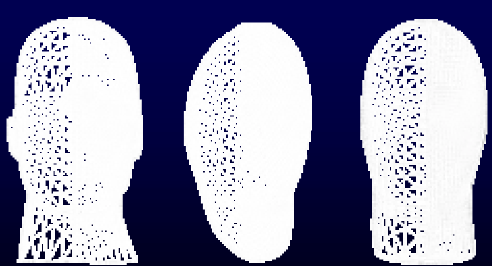


After derivation:

$$\kappa\,\mathbf{n} = \frac{1}{4A}\sum_j (\cot\alpha_j + \cot\beta_j)(\mathbf{P} - \mathbf{Q}_j)$$

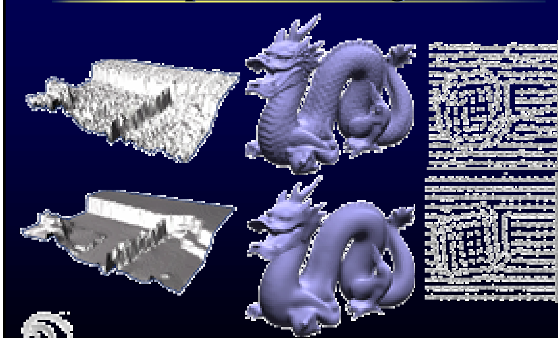
- Dependent on angles and lengths
- Zero curvature ensured when flat

Regular/Accurate Diffusion

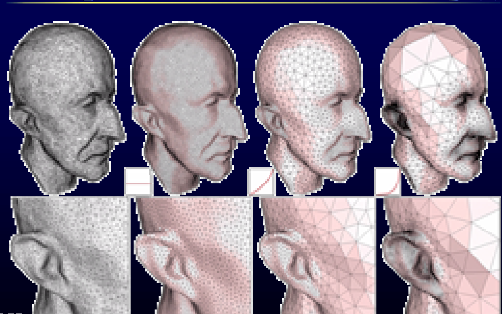


Initial mesh   Regular fairing   Accurate fairing

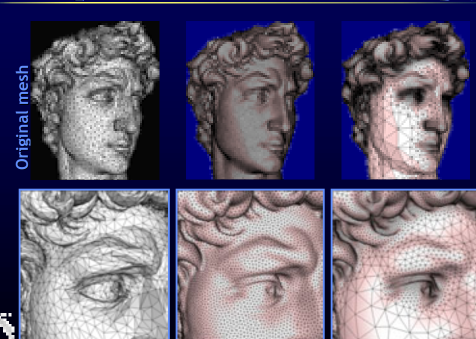
Anisotropic Smoothing Results



Example: Interactive Remeshing (I)

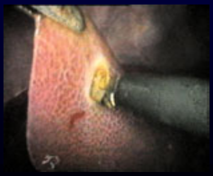
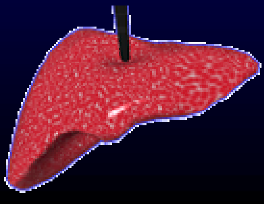


Example: Interactive Remeshing (II)



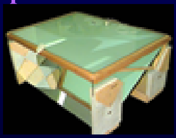
Animation: Virtual Simulators

- ✓ Virtual laparoscopic surgery
- ✓ Real time dynamic simulation (tough constraint!)
- ✓ Surface mesh deformation (new physical models)
- ✓ Force feedback

Test-bed for Virtual Reality

Responsive Workbench




Semi-immersive environment

Different current applications include
- medical & scientific visualization (MRI data, fluid flow, satellites, …)
- 3D art (3D painting, 3D sketching)
- pool anyone?

Movies

- First, a virtual actor
- Introducing… a real hat
- Same thing, in slow mo
- Discrete hat, same behavior
- Again, in slow mo
- A close up, and in slow mo
- A quite rigid hat, and in slow mo
- A floppy one, and in slow mo

Idea Takeaway

Math matters, too!
- DGP = more than just a bunch of triangles
- Need for powerful tools
- Difficulties
  - new generalizations
  - curved geometry
  - non-linear
  - inter-disciplinary collaborations

Only the Beginning

Digital Geometry Processing:
- emerging discipline
- process large geometry
  - acquisition, compression, editing, simulation,...
- mathematical foundations
  - Discrete Exterior Calculus
  - Discrete Differential Geometry
  - Discrete Mechanics for simulation

If you want to know more

http://www-grail.usc.edu/

http://www.multires.caltech.edu/

http://www.gg.caltech.edu/

http://www.siggraph.org/

http://www.usc.edu/dept/CGIT