Logical reasoning systems

- Theorem provers and logic programming languages
- Production systems
- Frame systems and semantic networks
- Description logic systems
Logical reasoning systems

- **Theorem provers and logic programming languages** – Provers: use resolution to prove sentences in full FOL. Languages: use backward chaining on restricted set of FOL constructs.
- **Production systems** – based on implications, with consequents interpreted as action (e.g., insertion & deletion in KB). Based on forward chaining + conflict resolution if several possible actions.
- **Frame systems and semantic networks** – objects as nodes in a graph, nodes organized as taxonomy, links represent binary relations.
- **Description logic systems** – evolved from semantic nets. Reason with object classes & relations among them.
Basic tasks

• Add a new fact to KB – TELL

• Given KB and new fact, derive facts implied by conjunction of KB and new fact. In forward chaining: part of TELL

• Decide if query entailed by KB – ASK

• Decide if query explicitly stored in KB – restricted ASK

• Remove sentence from KB: distinguish between correcting false sentence, forgetting useless sentence, or updating KB re. change in the world.
Indexing, retrieval & unification

- **Implementing sentences & terms**: define syntax and map sentences onto machine representation.

  **Compound**: has operator & arguments.
  
  e.g., \( c = P(x) \land Q(x) \)  
  \( \text{Op}[c] = \land; \text{Args}[c] = [P(x), Q(x)] \)

- **FETCH**: find sentences in KB that have same structure as query. ASK makes multiple calls to FETCH.

- **STORE**: add each conjunct of sentence to KB. Used by TELL.
  
  e.g., implement KB as list of conjuncts
  
  \( \text{TELL(KB, } A \land \neg B) \text{ TELL(KB, } \neg C \land D) \)  
  then KB contains: \([A, \neg B, \neg C, D]\)
Complexity

• With previous approach,

  FETCH takes $O(n)$ time on n-element KB

  STORE takes $O(n)$ time on n-element KB (if check for duplicates)

Faster solution?
Table-based indexing

• What are you indexing on? Predicates (relations/functions).
  Example:

<table>
<thead>
<tr>
<th>Key</th>
<th>Positive</th>
<th>Negative</th>
<th>Conclusion</th>
<th>Premise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mother</td>
<td>Mother(ann,sam)</td>
<td>-Mother(ann,al)</td>
<td>xxxx</td>
<td>xxxx</td>
</tr>
<tr>
<td></td>
<td>Mother(grace,joe)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dog</td>
<td>dog(rover)</td>
<td>-dog(alice)</td>
<td>xxxx</td>
<td>xxxx</td>
</tr>
<tr>
<td></td>
<td>dog(fido)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table-based indexing

- Use hash table to avoid looping over entire KB for each TELL or FETCH

  e.g., if only allowed literals are single letters, use a 26-element array to store their values.

- More generally:
  - convert to Horn form
  - index table by predicate symbol
  - for each symbol, store:
    - list of positive literals
    - list of negative literals
    - list of sentences in which predicate is in conclusion
    - list of sentences in which predicate is in premise
Tree-based indexing

- Hash table impractical if many clauses for a given predicate symbol

- Tree-based indexing (or more generally combined indexing): compute indexing key from predicate and argument symbols
Tree-based indexing

Example:

Person(age, height, weight, income)
Person(30, 72, 210, 45000)
Fetch(Person(age, 72, 210, income))
Fetch(Person(age, height > 72, weight < 210, income))
Unification algorithm: Example

Understands(mary,x) implies Loves(mary,x)
Understands(mary,pete) allows the system to substitute pete for x and make the implication that IF
Understands(mary,pete) THEN Loves(mary,pete)
Unification algorithm

• Using clever indexing, can reduce number of calls to unification

• Still, unification called very often (at basis of modus ponens) => need efficient implementation.

• See AIMA p. 303 for example of algorithm with $O(n^2)$ complexity
  (n being size of expressions being unified).
Logic programming

Remember: knowledge engineering vs. programming...

Sound bite: computation as inference on logical KBs

<table>
<thead>
<tr>
<th>Logic programming</th>
<th>Ordinary programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Identify problem</td>
<td>Identify problem</td>
</tr>
<tr>
<td>2. Assemble information</td>
<td>Assemble information</td>
</tr>
<tr>
<td>3. Tea break</td>
<td>Figure out solution</td>
</tr>
<tr>
<td>4. Encode information in KB</td>
<td>Program solution</td>
</tr>
<tr>
<td>5. Encode problem instance as facts</td>
<td>Encode problem instance as data</td>
</tr>
<tr>
<td>6. Ask queries</td>
<td>Apply program to data</td>
</tr>
<tr>
<td>7. Find false facts</td>
<td>Debug procedural errors</td>
</tr>
</tbody>
</table>

Should be easier to debug $Capital(\text{New York, US})$ than $x := x + 2$!
Logic programming systems

e.g., Prolog:

• Program = sequence of sentences (implicitly conjoined)
• All variables implicitly universally quantified
• Variables in different sentences considered distinct
• Horn clause sentences only (= atomic sentences or sentences with no negated antecedent and atomic consequent)
• Terms = constant symbols, variables or functional terms
• Queries = conjunctions, disjunctions, variables, functional terms
• Instead of negated antecedents, use negation as failure operator: goal NOT P considered proved if system fails to prove P
• Syntactically distinct objects refer to distinct objects
• Many built-in predicates (arithmetic, I/O, etc)
Prolog systems

Basis: backward chaining with Horn clauses + bells & whistles
Widely used in Europe, Japan (basis of 5th Generation project)
Compilation techniques ⇒ 10 million LIPS

Program = set of clauses = head :- literal₁, ... literalₙ.
Efficient unification by open coding
Efficient retrieval of matching clauses by direct linking
Depth-first, left-to-right backward chaining
Built-in predicates for arithmetic etc., e.g., X is Y*Z+3
Closed-world assumption ("negation as failure")
  e.g., not PhD(X) succeeds if PhD(X) fails
Basic syntax of facts, rules and queries

<fact> ::= <term> .

<rule> ::= <term> :- <term> .

<query> ::= <term> .

<term> ::= <number> | <atom> | <variable>
        | <atom> (<terms>)

<terms> ::= <term> | <term>, <terms>
A PROLOG program is a set of **facts** and **rules**.

A simple program with just facts:

```
parent(alice, jim).
parent(jim, tim).
parent(jim, dave).
parent(jim, sharon).
parent(tim, james).
parent(tim, thomas).
```
A PROLOG Program

- c.f. a table in a relational database.
- Each line is a **fact** (a.k.a. a tuple or a row).
- Each line states that some person \( x \) is a parent of some (other) person \( y \).
- In GNU PROLOG the program is kept in an ASCII file.
• Now we can ask PROLOG questions:
  |  ?- parent(alice, jim).
  yes
  |  ?- parent(jim, herbert).
  no
  |  ?-
A PROLOG Query

• Not very exciting. But what about this:

| ?- parent(alice, Who).
| Who = jim
| yes
| ?-

• Who is called a **logical variable**.

• PROLOG will set a logical variable to any value which makes the query succeed.
• Sometimes there is more than one correct answer to a query.

• PROLOG gives the answers one at a time. To get the next answer type ;.

| ?- parent(jim, Who).  
| Who = tim ? ;  
| Who = dave ? ;  
| Who = sharon ? ;  
| yes  
| ?-  

NB : The ; do not actually appear on the screen.
A PROLOG Query II

| ?- parent(jim, Who).
Who = tim ? ;
Who = dave ? ;
Who = sharon ? ;
yes
| ?-  

- After finding that jim was a parent of sharon GNU PROLOG detects that there are no more alternatives for parent and ends the search.

NB: The ; do not actually appear on the screen.
Prolog example

Depth-first search from a start state X:

dfs(X) :- goal(X).
dfs(X) :- successor(X,S); dfs(S).

No need to loop over S: successor succeeds for each

Appending two lists to produce a third:

append([],Y,Y).
append([X|L],Y,[X|Z]) :- append(L,Y,Z).

query: append(A,B,[1,2]) ?
answers: A=[] B=[1,2]
A=[1,2] B=[]
Append

- \texttt{append([], L, L)}
- \texttt{append([H| L1], L2, [H| L3]) :- append(L1, L2, L3)}

- Example join \([a, b, c]\) with \([d, e]\).
  - \([a, b, c]\) has the recursive structure \([a| [b, c]]\).
  - Then the rule says:
    - IF \([b, c]\) appends with \([d, e]\) to form \([b, c, d, e]\) THEN \([a| [b, c]]\) appends with \([d, e]\) to form \([a| [b, c, d, e]]\)
  - i.e. \([a, b, c]\)                \([a, b, c, d, e]\)
Expanding Prolog

- **Parallelization:**
  - OR-parallelism: goal may unify with many different literals and implications in KB
  - AND-parallelism: solve each conjunct in body of an implication in parallel

- **Compilation:** generate built-in theorem prover for different predicates in KB

- **Optimization:** for example through re-ordering
  - e.g., “what is the income of the spouse of the president?”
    \[
    \text{Income}(s, i) \land \text{Married}(s, p) \land \text{Occupation}(p, \text{President})
    \]
  faster if re-ordered as:
  \[
  \text{Occupation}(p, \text{President}) \land \text{Married}(s, p) \land \text{Income}(s, i)
  \]
Theorem provers

- Differ from logic programming languages in that:
  - accept full FOL
  - results independent of form in which KB entered
OTTER

- Organized Techniques for Theorem Proving and Effective Research (McCune, 1992)

- **Set of support (sos):** set of clauses defining facts about problem
- Each resolution step: resolves member of sos against other axiom
- **Usable axioms** (outside sos): provide background knowledge about domain
- **Rewrites** (or demodulators): define canonical forms into which terms can be simplified. E.g., \( x+0=x \)
- **Control strategy:** defined by set of parameters and clauses. E.g., heuristic function to control search, filtering function to eliminate uninteresting subgoals.
OTTER

• Operation: resolve elements of sos against usable axioms

• Use best-first search: heuristic function measures “weight” of each clause (lighter weight preferred; thus in general weight correlated with size/difficulty)

• At each step: move lightest close in sos to usable list, and add to usable list consequences of resolving that close against usable list

• Halt: when refutation found or sos empty
Otter: An Automated Deduction System


Contents

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3. Availability
4. Documentation
5. Example Inputs
6. Recent Accomplishments
7. Performance on the TPTP Problems
8. Bugs and Fixes
9. Otter users Mailing List

Related Pages

- Try Otter right now with Son of BirdBrain
- A sample Otter proof
- New Results obtained with Otter and related programs
- MACE, a program that searches for small models
- ERF, a prover for equational logic with associative unification
- Automated Reasoning at Argonne

External Work

- Johan Belinfante’s Set Theory Work with Otter
- Some other theorem provers
- Otter made for Irvine (from Holger Schau"
- GOAL, by Guoqiang Huang and Dale Myers
- A student project on Otter by Jackson Paul

Description

Our current automated deduction system Otter is designed to prove theorems stated in first-order logic with equality. Otter’s inference rules are based on resolution and paramodulation, and it includes facilities for term rewriting, term orderings, Knuth-Bendix completion, weighting, and strategies for directing
Example: Robbins Algebras Are Boolean

- The Robbins problem---are all Robbins algebras Boolean?---has been solved: Every Robbins algebra is Boolean. This theorem was proved automatically by EQP, a theorem proving program developed at Argonne National Laboratory.
Example: Robbins Algebras Are Boolean

Historical Background

• In 1933, E. V. Huntington presented the following basis for Boolean algebra:

\[ x + y = y + x. \quad [\text{commutativity}] \]
\[ (x + y) + z = x + (y + z). \quad [\text{associativity}] \]
\[ n(n(x) + y) + n(n(x) + n(y)) = x. \quad [\text{Huntington equation}] \]

• Shortly thereafter, Herbert Robbins conjectured that the Huntington equation can be replaced with a simpler one:

\[ n(n(x + y) + n(x + n(y))) = x. \quad [\text{Robbins equation}] \]

• Robbins and Huntington could not find a proof, and the problem was later studied by Tarski and his students
Searching ...

Success, in 1.28 seconds!

------------- PROOF -------------

1  n(n(A)+n(B))+n(n(A)+n(B))!=A.
2    x=x.                     [para_from, 3, 1]
3  x+y=y+x.                  [para_into, 4, 8, flip.1]
5, 4 (x+y)+z=x+(y+z).        [para_into, 4, 3, demod.5]
6  n(n(x+y)+n(x+n(y)))=x.    [para_into, 13, 3]
8  x+x=x.                    [para_into, 6, 8]
10 n(n(A)+n(B))+n(n(A)+B)!=A. [para_into, 6, 8]
13  x+ (x+y)=x+y.            [para_into, 6, 3]
15  x+ (y+z)=y+ (x+z).       [para_into, 15, 3, demod.5]
23, 22 x+ (y+z)=x+y.         [para_into, 26, 3]
26  n(n(x)+n(x+n(x)))=x.    [para_into, 26, 6, demod, 23]
36  n(n(n(x)+x)+n(n(x)))=n(x). [para_into, 26, 6, demod, 23]
42  n(n(x)+n(y))+n(n(x+y))=x. [para_from, 80, 80, demod, 5, 81]
52  x+ (y+z)=z+(z+y).        [para_into, 80, 80, demod, 5, 81]
81, 80 n(n(x+n(x))+n(x))=x.  [para_into, 82, 3]
82  n(n(x)+x)+x)=n(x+n(x)).  [para_into, 139, demod, 166]
125 n(n(n(x)+x)+x)+x)=n(x+n(x))+n(x). [para_into, 165, 165, demod, 5, 180, 5, 166]
139 n(n(x+n(x))+x)+n(x)=n(x+n(x)). [para_into, 165, 165, demod, 5, 180, 5, 166]
166, 165 n(n(x+n(x))+x)=n(x). [para_into, 165, 80, demod, 166, 5, 180, 5]
180, 179 n(n(x)+x)=n(x+n(x)). [para_into, 179, 52, demod, 5]
195 n(n(x)+n(x))+n(n(x)))=n(x). [back_demod, 125, demod, 223]
197 n(n(x+ (n(x)+n(x+((x)))))+ (n(x+n(x))+x))=n(x). [back_demod, 125, demod, 223]
206, 205 n(n(x+ (n(x)+n(x+n(x))))+n(x)))=n(x+n(x))+x. [back_demod, 197, demod, 584, 231]
223, 222 n(n(x)+y)+ (y+x))=n(x+ (y+y)). [para_into, 585, 42, flip.1]
231, 230 n(n(x)+n(x+n(x))+x)=n(x+n(x))+n(x). [binary, 621, 2]
564, 563 n(x+n(x))+x=x.      [para_into, 585, 42, flip.1]
582, 581 n(x+n(x))+n(x)=n(x). [back_demod, 584, 231]
586, 585 n(n(x))=x.         [back_demod, 80, demod, 582]
606, 605 n(x+n(y))+n(x+y)=n(x). [back_demod, 10, demod, 606, 586]
621 A!=A.                    [binary, 621, 2]
622 $F$.
Forward-chaining production systems

- Prolog & other programming languages: rely on backward-chaining
  (I.e., given a query, find substitutions that satisfy it)

- Forward-chaining systems: infer everything that can be inferred from KB each time new sentence is TELL’ed

- Appropriate for agent design: as new percepts come in, forward-chaining returns best action
Implementation

- One possible approach: use a theorem prover, using resolution to forward-chain over KB

- More restricted systems can be more efficient.

- Typical components:
  - KB called “working memory” (positive literals, no variables)
  - rule memory (set of inference rules in form
    \[ p_1 \land p_2 \land \ldots \Rightarrow \text{act}_1 \land \text{act}_2 \land \ldots \]
  - at each cycle: find rules whose premises satisfied by working memory (match phase)
  - decide which should be executed (conflict resolution phase)
  - execute actions of chosen rule (act phase)
Match phase

- Unification can do it, but inefficient

- Rete algorithm (used in OPS-5 system): example

  rule memory:
  
  \[ A(x) \land B(x) \land C(y) \Rightarrow add \ D(x) \]
  
  \[ A(x) \land B(y) \land D(x) \Rightarrow add \ E(x) \]
  
  \[ A(x) \land B(x) \land E(x) \Rightarrow delete \ A(x) \]

  working memory:
  
  \{A(1), A(2), B(2), B(3), B(4), C(5)\}

- Build Rete network from rule memory, then pass working memory through it
Rete network

Circular nodes: fetches to WM; rectangular nodes: unifications

\[ A(x) \land B(x) \land C(y) \Rightarrow \text{add } D(x) \]
\[ A(x) \land B(y) \land D(x) \Rightarrow \text{add } E(x) \]
\[ A(x) \land B(x) \land E(x) \Rightarrow \text{delete } A(x) \]

\{A(1), A(2), B(2), B(3), B(4), C(5)\}
Rete match

\[ A(x) \land B(x) \land C(y) \Rightarrow \text{add } D(x) \]
\[ A(x) \land B(y) \land D(x) \Rightarrow \text{add } E(x) \]
\[ A(x) \land B(x) \land E(x) \Rightarrow \text{delete } A(x) \]
Advantages of Rete networks

• Share common parts of rules

• Eliminate duplication over time (since for most production systems only a few rules change at each time step)
Conflict resolution phase

- One strategy: execute all actions for all satisfied rules
- Or, treat them as suggestions and use conflict resolution to pick one action.

Strategies:
- No duplication (do not execute twice same rule on same args)
- Regency (prefer rules involving recently created WM elements)
- Specificity (prefer more specific rules)
- Operation priority (rank actions by priority and pick highest)
Frame systems & semantic networks

• Other notation for logic; equivalent to sentence notation

• Focus on categories and relations between them (remember ontologies)

  \[ \text{Subset} \]

• e.g., Cats \rightarrow Mammals
## Syntax and Semantics

<table>
<thead>
<tr>
<th>Link Type</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \subseteq B$</td>
<td>$A \subseteq B$</td>
</tr>
<tr>
<td>$A \in B$</td>
<td>$A \in B$</td>
</tr>
<tr>
<td>$R(A,B)$</td>
<td>$R(A,B)$</td>
</tr>
<tr>
<td>$\forall x \in A \Rightarrow R(x,y)$</td>
<td>$\forall x \in A \Rightarrow R(x,y)$</td>
</tr>
<tr>
<td>$\forall x \exists y \in B \wedge R(x,y)$</td>
<td>$\forall x \exists y \in B \wedge R(x,y)$</td>
</tr>
</tbody>
</table>
Semantic Network Representation

Animal
  has
  can

Skin
  can
  has

Move
  can

Fish
  has

Wings
  has

Feathers

Bird
  has
  can
  Is a

Canary
  has
  is

Ostrich
  is
  cannot

Sing
  is

Yellow
  Fly

Fly
  Tall

CS 460, Session 19
# Semantic network link types

<table>
<thead>
<tr>
<th>Link type</th>
<th>Semantics</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Subset</em></td>
<td>$A \subseteq B$</td>
<td>Cats $\rightarrow$ Mammals</td>
</tr>
<tr>
<td><em>Member</em></td>
<td>$A \in B$</td>
<td>Bill $\rightarrow$ Cats</td>
</tr>
<tr>
<td>$R$</td>
<td>$R(A, B)$</td>
<td>Bill $\rightarrow$ 12</td>
</tr>
<tr>
<td>$R$</td>
<td>$\forall x \in A \Rightarrow R(x, B)$</td>
<td>Birds $\rightarrow$ 2</td>
</tr>
<tr>
<td>$R$</td>
<td>$\forall x \exists y \in A \Rightarrow y \in B \land R(x, y)$</td>
<td>Birds $\rightarrow$ Birds</td>
</tr>
</tbody>
</table>