6.189 IAP 2007

Lecture 11

Parallelizing Compilers

Outline

- **Parallel Execution**
- Parallelizing Compilers
- Dependence Analysis
- Increasing Parallelization Opportunities
- Generation of Parallel Loops
- Communication Code Generation

Types of Parallelism

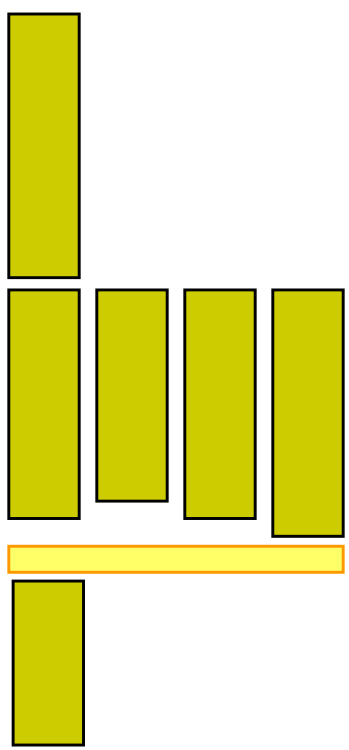
- Instruction Level Parallelism (ILP) → Scheduling and Hardware
- Task Level Parallelism (TLP) → Mainly by hand
- Loop Level Parallelism (LLP) or Data Parallelism → Hand or Compiler Generated
- Pipeline Parallelism → Hardware or Streaming
- Divide and Conquer Parallelism → Recursive functions

Why Loops?

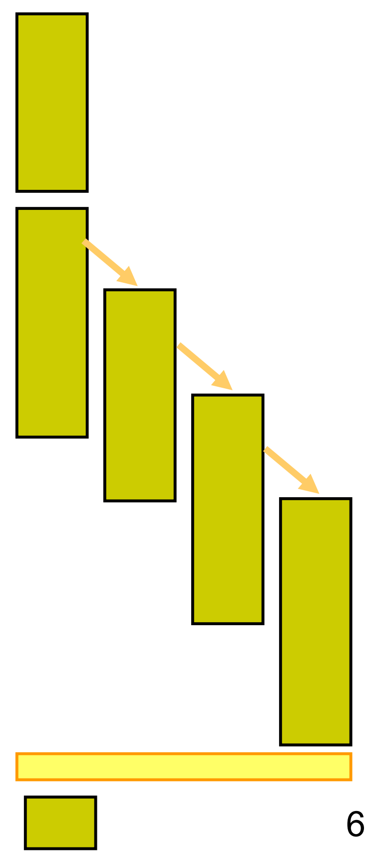
- 90% of the execution time in 10% of the code
  - Mostly in loops
- If parallel, can get good performance
  - Load balancing
- Relatively easy to analyze

Programmer Defined Parallel Loop

- FORALL
  - No "loop carried dependences"
  - Fully parallel



- FORACROSS
  - Some "loop carried dependences"



Parallel Execution

- Example

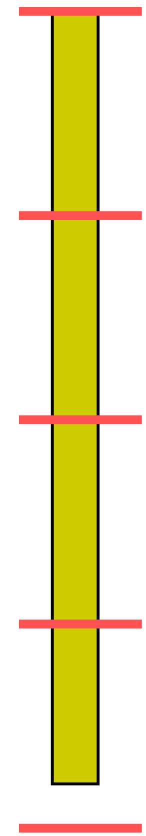
```
FORPAR I = 0 to N
   A[I] = A[I] + 1
```

- Block Distribution: Program gets mapped into

```
Iters = ceiling(N/NUMPROC);
FOR P = 0 to NUMPROC-1
   FOR I = P*Iters to MIN((P+1)*Iters, N)
     A[I] = A[I] + 1
```

- SPMD (Single Program, Multiple Data) Code

```
If(myPid == 0) {
   …
   Iters = ceiling(N/NUMPROC);
}
Barrier();
FOR I = myPid*Iters to MIN((myPid+1)*Iters, N)
   A[I] = A[I] + 1
Barrier();
```



Parallel Execution

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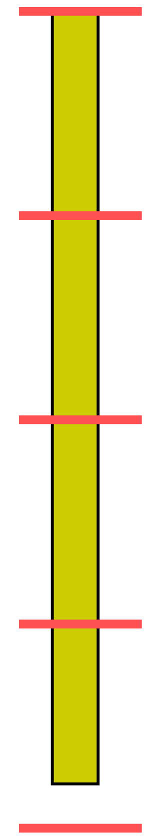
FOR P = 0 to NUMPROC-1
   FOR I = P*Iters to MIN((P+1)*Iters, N)
     A[I] = A[I] + 1
```

- Code that fork a function

```
Iters = ceiling(N/NUMPROC);
ParallelExecute(func1);
…
void func1(integer myPid)
{
   FOR I = myPid*Iters to MIN((myPid+1)*Iters, N)
     A[I] = A[I] + 1
}
```



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- **Parallelizing Compilers**
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Parallelizing Compilers

- Finding FORALL Loops out of FOR loops

- Examples

```
FOR I = 0 to 5
   A[I+1] = A[I] + 1

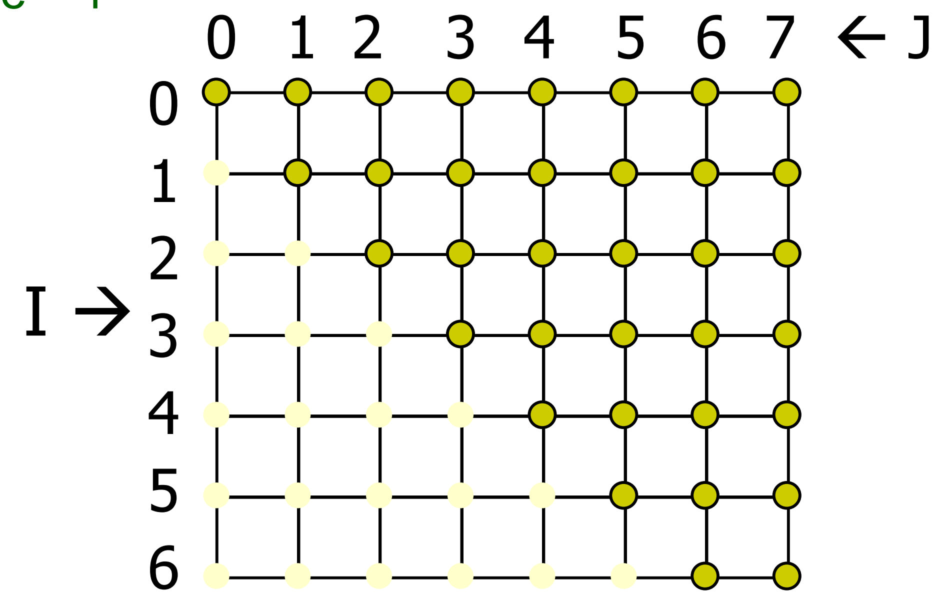
FOR I = 0 to 5
   A[I] = A[I+6] + 1

For I = 0 to 5
   A[2*I] = A[2*I + 1] + 1
```

Iteration Space

- N deep loops → n-dimensional discrete cartesian space
  - Normalized loops: assume step size = 1

```
FOR I = 0 to 6
   FOR J = I to 7
```

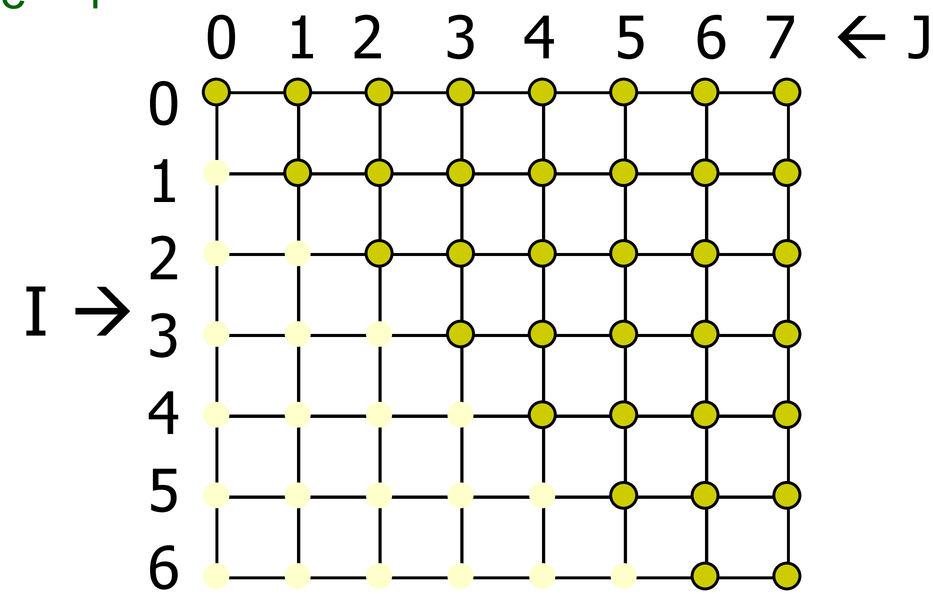


- Iterations are represented as coordinates in iteration space
  - $\bar{i} = [i_1, i_2, i_3, \ldots, i_n]$

Iteration Space

- N deep loops → n-dimensional discrete cartesian space
  - Normalized loops: assume step size = 1

```
FOR I = 0 to 6
   FOR J = I to 7
```



- Iterations are represented as coordinates in iteration space
- Sequential execution order of iterations
  ➔ Lexicographic order

  [0,0], [0,1], [0,2], …, [0,6], [0,7],
  [1,1], [1,2], …, [1,6], [1,7],
  [2,2], …, [2,6], [2,7],
  ………
  [6,6], [6,7],

Iteration Space

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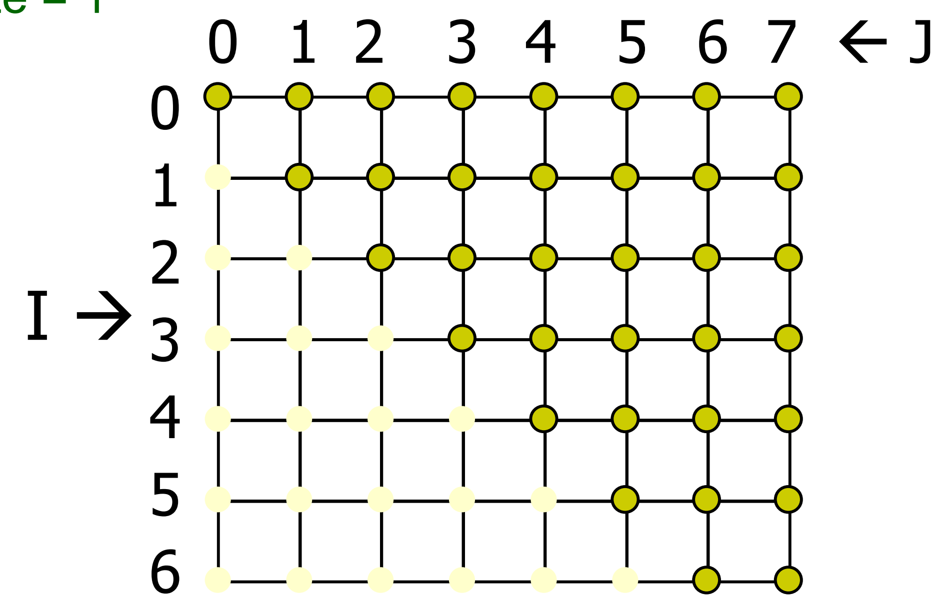
```
FOR I = 0 to 6
   FOR J = I to 7
```

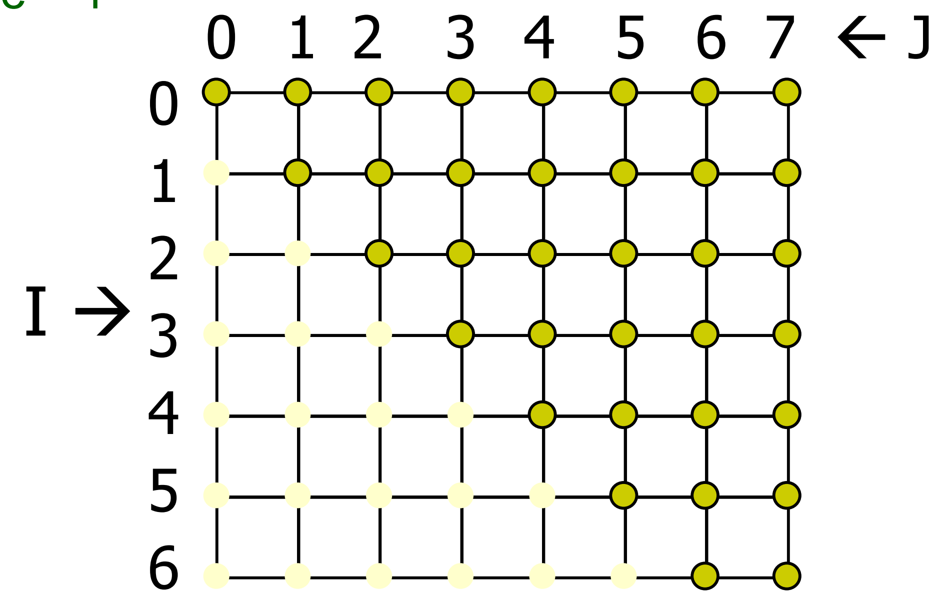


- Iterations are represented as coordinates in iteration space
- Sequential execution order of iterations
  ➔ Lexicographic order
- Iteration $\bar{i}$ is lexicograpically less than $\bar{j}$, $\bar{i} < \bar{j}$ iff
  there exists c s.t. $i_1 = j_1$, $i_2 = j_2$,… $i_{c-1} = j_{c-1}$ and $i_c < j_c$

Iteration Space

- N deep loops → n-dimensional discrete cartesian space
  - Normalized loops: assume step size = 1

```
FOR I = 0 to 6
   FOR J = I to 7
```

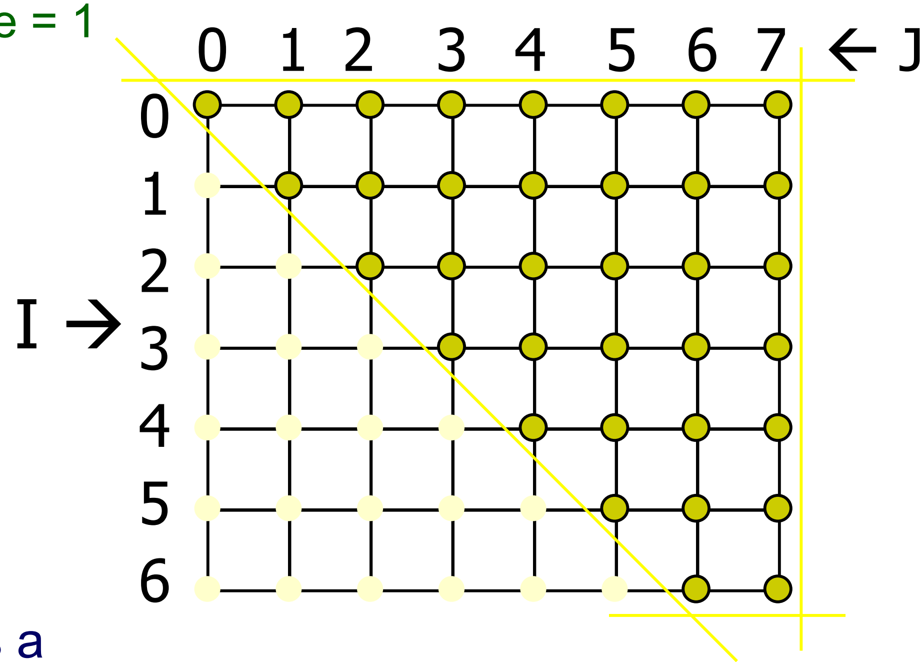


- An affine loop nest
  - Loop bounds are integer linear functions of constants, loop constant variables and outer loop indexes
  - Array accesses are integer linear functions of constants, loop constant variables and loop indexes

Iteration Space

- N deep loops → n-dimensional discrete cartesian space
  - Normalized loops: assume step size = 1

```
FOR I = 0 to 6
   FOR J = I to 7
```



- Affine loop nest → Iteration space as a set of liner inequalities

  $0 \leq I$

  $I \leq 6$

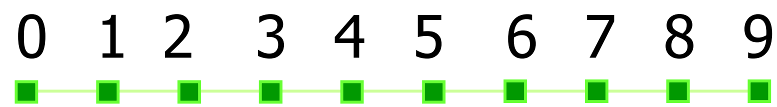
  $I \leq J$

  $J \leq 7$

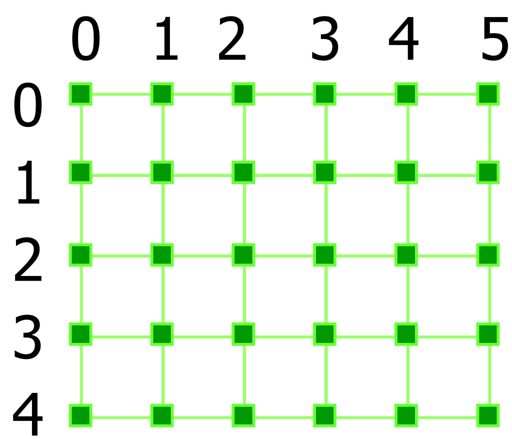
Data Space

- M dimensional arrays → m-dimensional discrete cartesian space
  - a hypercube

```
Integer A(10)
```




```
Float B(5, 6)
```



Dependences

- True dependence

```
a  =
   =  a
```

- Anti dependence

```
   =  a
a  =
```

- Output dependence

```
a  =
a  =
```

- Definition:
  Data dependence exists for a dynamic instance i and j iff
  - either i or j is a write operation
  - i and j refer to the same variable
  - i executes before j
- How about array accesses within loops?

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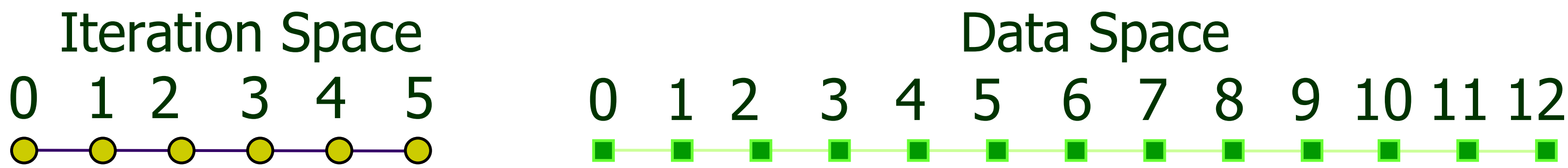
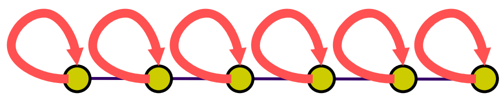
Array Accesses in a loop

```
FOR I = 0 to 5
  A[I] = A[I] + 1
```



Array Accesses in a loop



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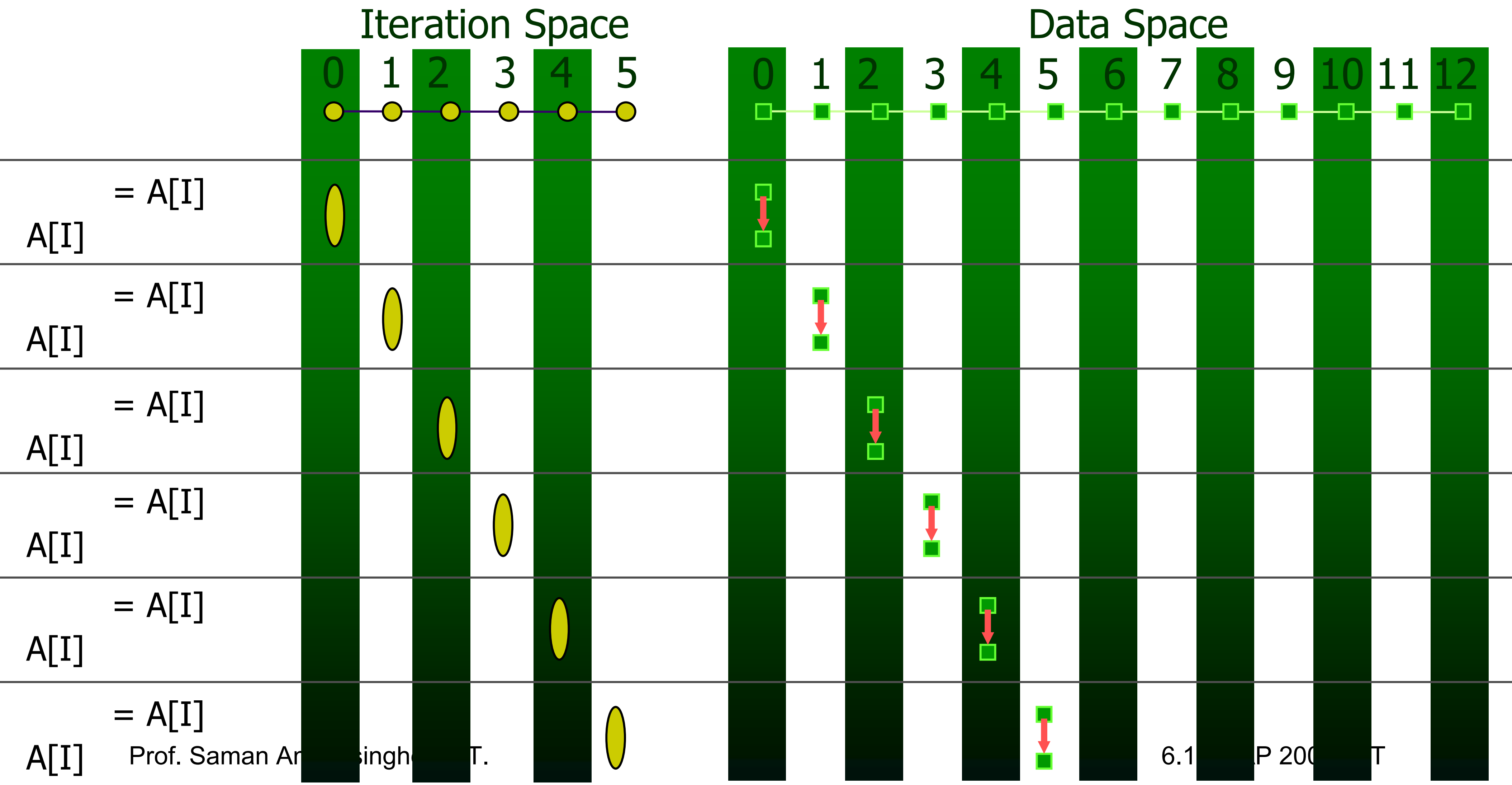
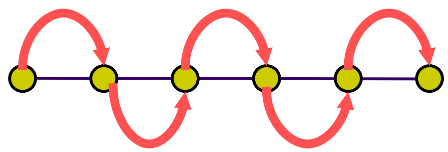
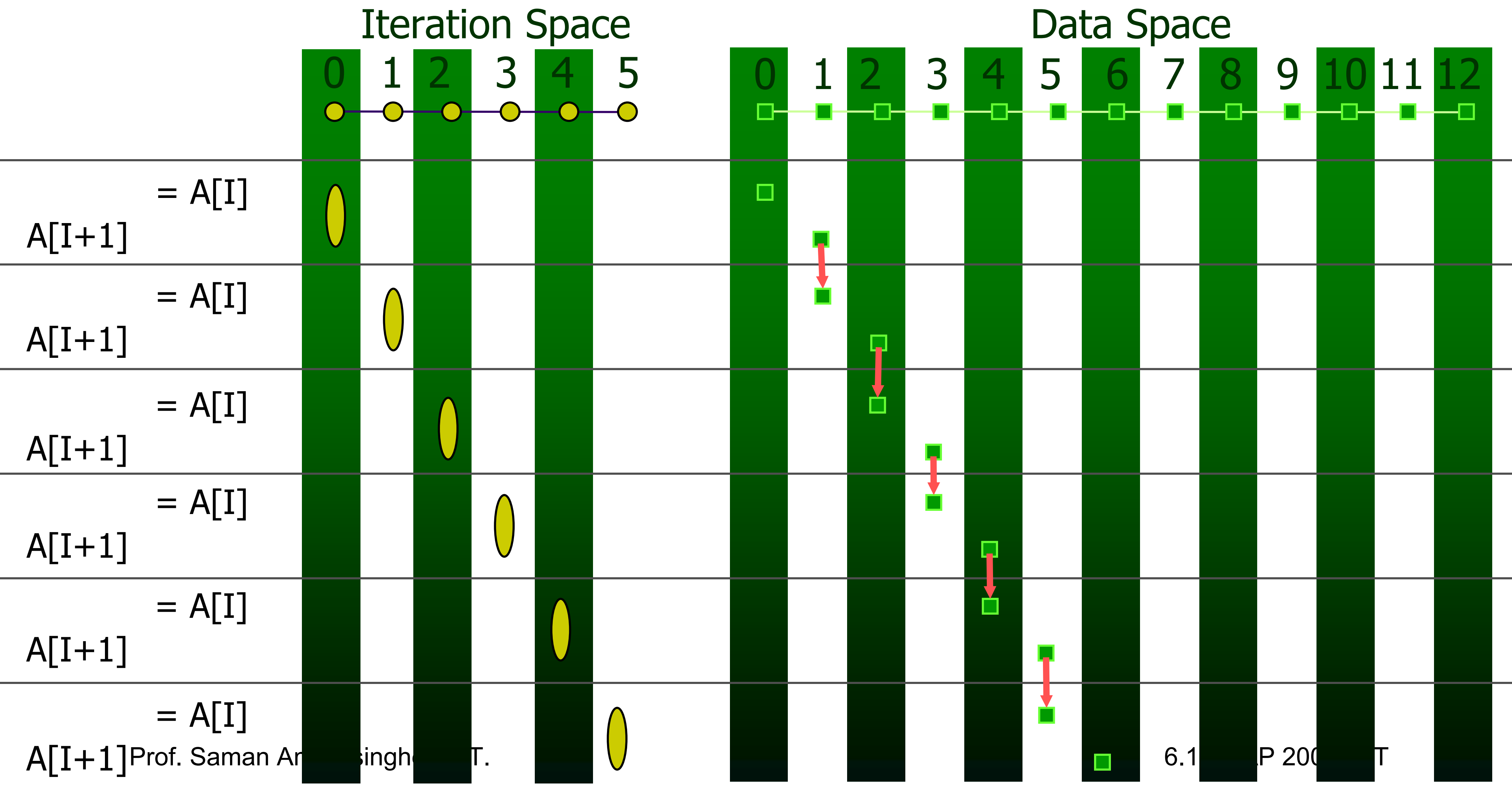


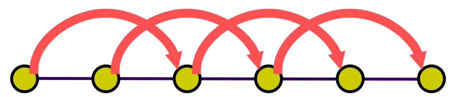
Array Accesses in a loop



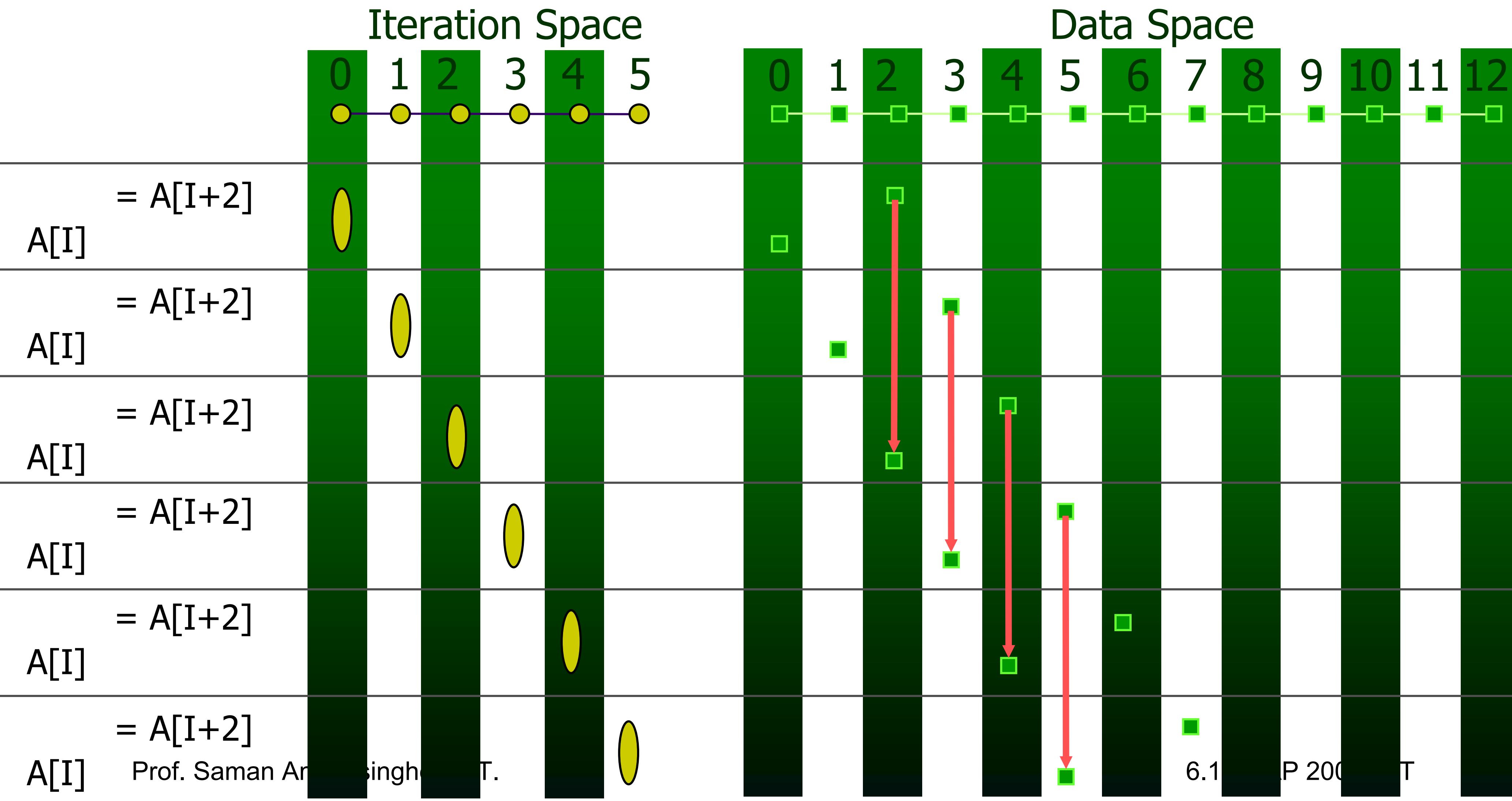
```
FOR I = 0 to 5
   A[I+1] = A[I] + 1
```



Array Accesses in a loop

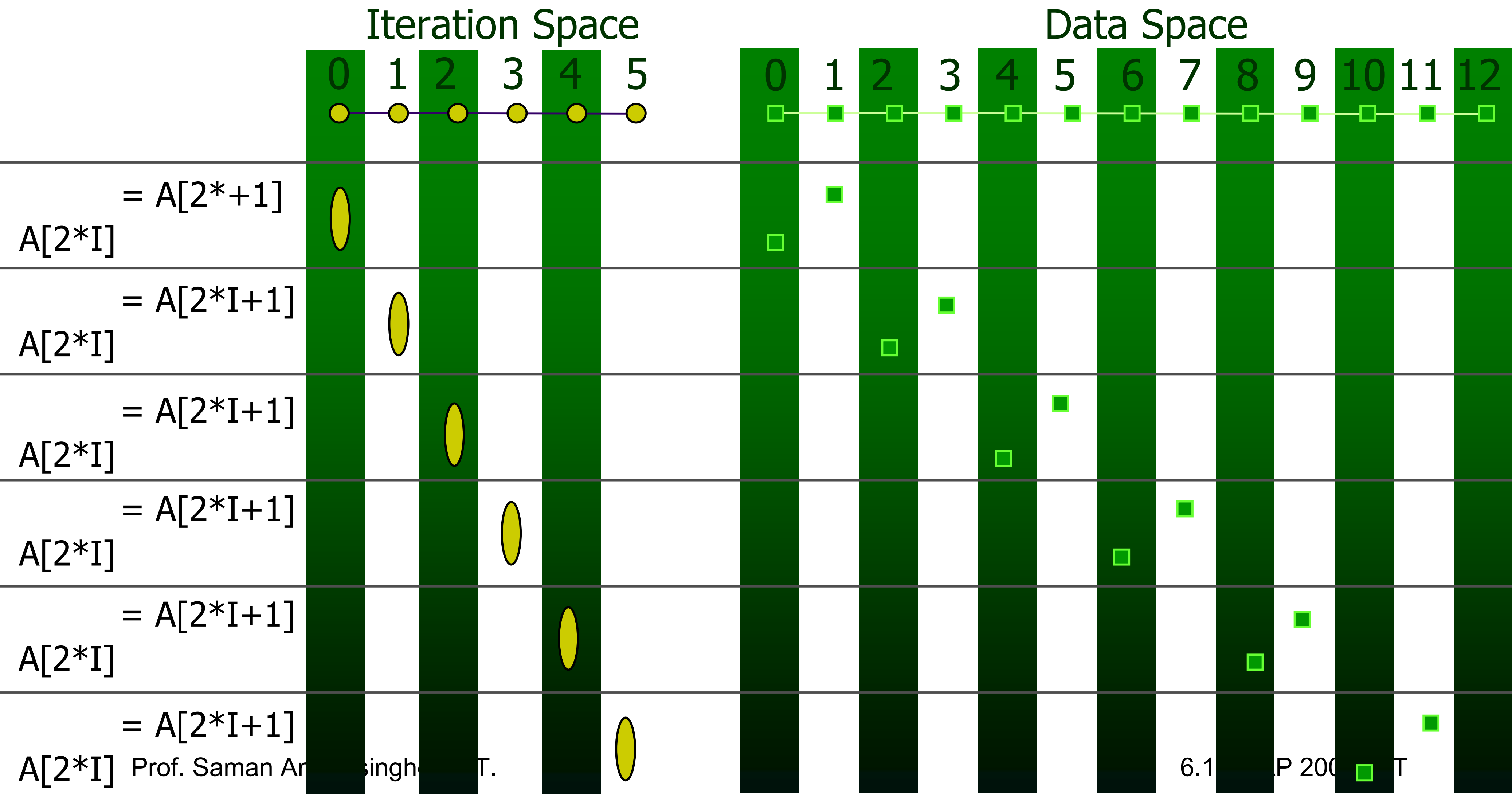


```
FOR I = 0 to 5
   A[I] = A[I+2] + 1
```



Array Accesses in a loop

```
FOR I = 0 to 5
   A[2*I] = A[2*I+1] + 1
```



Recognizing FORALL Loops

- Find data dependences in loop
  - For every pair of array acceses to the same array

    If the first access has at least one dynamic instance (an iteration) in which it refers to a location in the array that the second access also refers to in at least one of the later dynamic instances (iterations).

    Then there is a data dependence between the statements
  - (Note that same array can refer to itself – output dependences)

- Definition
  - Loop-carried dependence:
    dependence that crosses a loop boundary

- If there are no loop carried dependences → parallelizable

Data Dependence Analysis

- Example

```
FOR I = 0 to 5
   A[I+1] = A[I] + 1
```

- Is there a loop-carried dependence between A[I+1] and A[I]
  - Is there two distinct iterations i_w and i_r such that $A[i_w+1]$ is the same location as $A[i_r]$
  - $\exists$ integers i_w, i_r $\quad 0 \leq i_w, i_r \leq 5 \quad i_w \neq i_r \quad i_w + 1 = i_r$

- Is there a dependence between A[I+1] and A[I+1]
  - Is there two distinct iterations i_1 and i_2 such that $A[i_1+1]$ is the same location as $A[i_2+1]$
  - $\exists$ integers i_1, i_2 $\quad 0 \leq i_1, i_2 \leq 5 \quad i_1 \neq i_2 \quad i_1 + 1 = i_2 + 1$

Integer Programming

- Formulation
  - ∃ an integer vector $\bar{i}$ such that $\hat{A}\,\bar{i} \le \bar{b}$ where $\hat{A}$ is an integer matrix and $\bar{b}$ is an integer vector
- Our problem formulation for A[i] and A[i+1]
  - ∃ integers i_w, i_r $0 \le i_w, i_r \le 5$ $i_w \ne i_r$ $i_w + 1 = i_r$
  - $i_w \ne i_r$ is not an affine function
    - divide into 2 problems
    - Problem 1 with $i_w < i_r$ and problem 2 with $i_r < i_w$
    - If either problem has a solution → there exists a dependence
  - How about $i_w + 1 = i_r$
    - Add two inequalities to single problem
      $i_w + 1 \le i_r$, and $i_r \le i_w + 1$

Integer Programming Formulation

- Problem 1

  $0 \leq i_w$

  $i_w \leq 5$

  $0 \leq i_r$

  $i_r \leq 5$

  $i_w < i_r$

  $i_w + 1 \leq i_r$

  $i_r \leq i_w + 1$

Integer Programming Formulation

- Problem 1

$0 \le i_w$	→	$-i_w \le 0$
$i_w \le 5$	→	$i_w \le 5$
$0 \le i_r$	→	$-i_r \le 0$
$i_r \le 5$	→	$i_r \le 5$
$i_w < i_r$	→	$i_w - i_r \le -1$
$i_w + 1 \le i_r$	→	$i_w - i_r \le -1$
$i_r \le i_w + 1$	→	$-i_w + i_r \le 1$

Integer Programming Formulation

- Problem 1

Constraint		Rewritten	$\hat{A}$		$\bar{b}$
$0 \le i_w$	→	$-i_w \le 0$	-1	0	0
$i_w \le 5$	→	$i_w \le 5$	1	0	5
$0 \le i_r$	→	$-i_r \le 0$	0	-1	0
$i_r \le 5$	→	$i_r \le 5$	0	1	5
$i_w < i_r$	→	$i_w - i_r \le -1$	1	-1	-1
$i_w + 1 \le i_r$	→	$i_w - i_r \le -1$	1	-1	-1
$i_r \le i_w + 1$	→	$-i_w + i_r \le 1$	-1	1	1

$$\hat{A} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad \bar{b} = \begin{pmatrix} 0 \\ 5 \\ 0 \\ 5 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

- and problem 2 with $i_r < i_w$

Generalization

- An affine loop nest

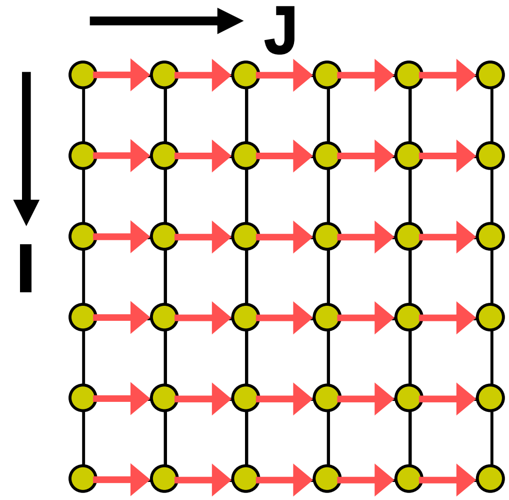
```
FOR i1 = fl1(c1…ck) to Iu1(c1…ck)
  FOR i2 = fl2(i1,c1…ck) to Iu2(i1,c1…ck)
    ……
     FOR in = fln(i1…in-1,c1…ck) to Iun(i1…in-1,c1…ck)
          A[fa1(i1…in,c1…ck), fa2(i1…in,c1…ck),…,fam(i1…in,c1…ck)]
```

- Solve 2*n problems of the form
  - $i_1 = j_1,\ i_2 = j_2, \ldots\ldots\ i_{n-1} = j_{n-1},\ i_n < j_n$
  - $i_1 = j_1,\ i_2 = j_2, \ldots\ldots\ i_{n-1} = j_{n-1},\ j_n < i_n$
  - $i_1 = j_1,\ i_2 = j_2, \ldots\ldots\ i_{n-1} < j_{n-1}$
  - $i_1 = j_1,\ i_2 = j_2, \ldots\ldots\ j_{n-1} < i_{n-1}$

  ………………….
  - $i_1 = j_1,\ i_2 < j_2$
  - $i_1 = j_1,\ j_2 < i_2$
  - $i_1 < j_1$
  - $j_1 < i_1$

Multi-Dimensional Dependence

```
FOR I = 1 to n
   FOR J = 1 to n
     A[I, J] = A[I, J-1] + 1
```



Multi-Dimensional Dependence

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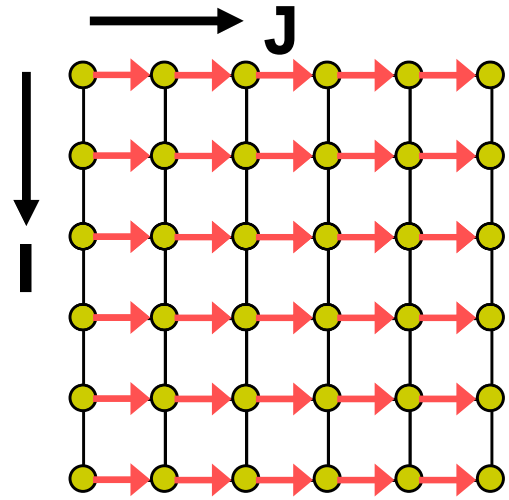
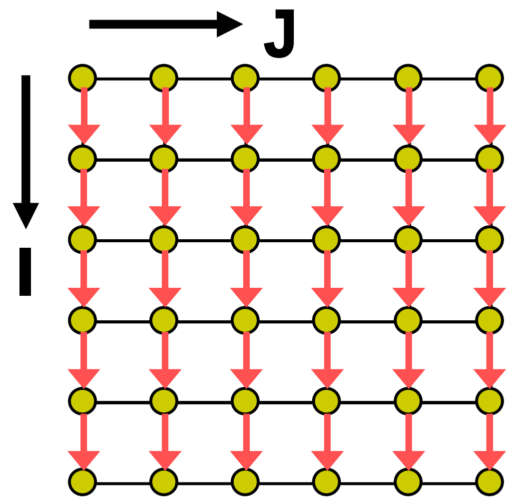
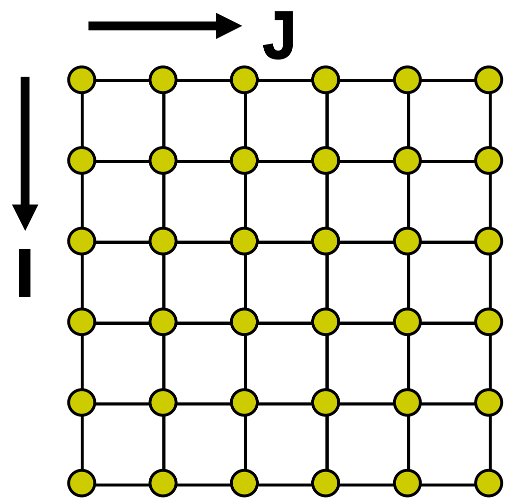



```
FOR I = 1 to n
   FOR J = 1 to n
      A[I, J] = A[I+1, J] + 1
```

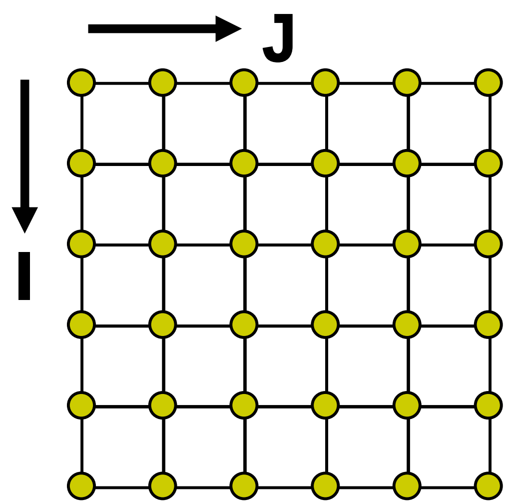


What is the Dependence?

```
FOR I = 1 to n
   FOR J = 1 to n
     A[I, J] = A[I-1, J+1] + 1
```

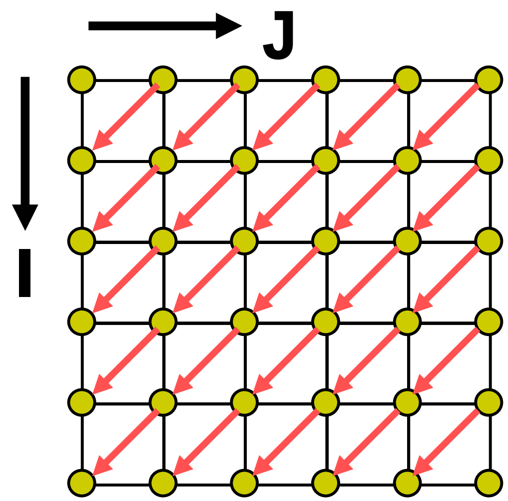



```
FOR I = 1 to n
   FOR J = 1 to n
     B[I] = B[I-1] + 1
```

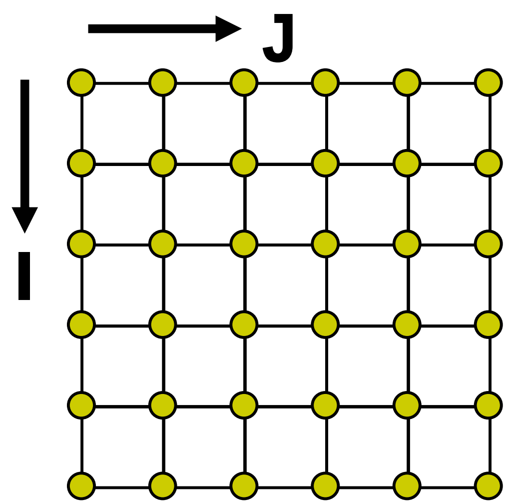


What is the Dependence?

```
FOR I = 1 to n
   FOR J = 1 to n
     A[I, J] = A[I-1, J+1] + 1
```

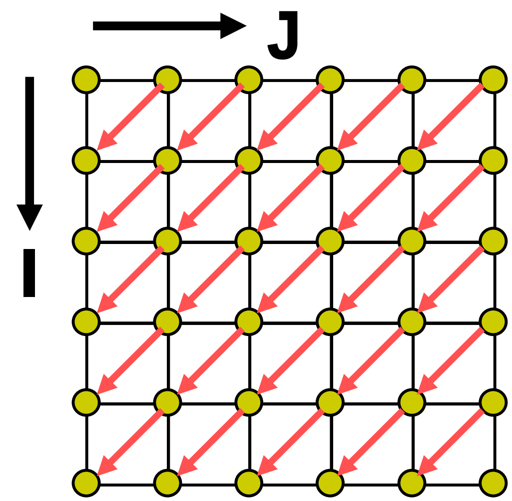



```
FOR I = 1 to n
   FOR J = 1 to n
     A[I] = A[I-1] + 1
```

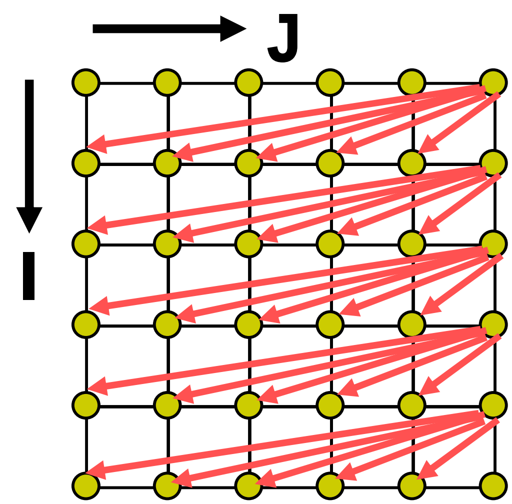


What is the Dependence?

```
FOR I = 1 to n
   FOR J = 1 to n
     A[I, J] = A[I-1, J+1] + 1
```




```
FOR I = 1 to n
   FOR J = 1 to n
     B[I] = B[I-1] + 1
```



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- Parallel Execution
- Parallelizing Compilers
- Dependence Analysis
- **Increasing Parallelization Opportunities**
- Generation of Parallel Loops
- Communication Code Generation

Increasing Parallelization Opportunities

- Scalar Privatization
- Reduction Recognition
- Induction Variable Identification
- Array Privatization
- Interprocedural Parallelization
- Loop Transformations
- Granularity of Parallelism

Scalar Privatization

- Example

```
FOR i = 1 to n
   X = A[i] * 3;
   B[i] = X;
```

- Is there a loop carried dependence?
- What is the type of dependence?

Privatization

- Analysis:
  - Any anti- and output- loop-carried dependences

- Eliminate by assigning in local context

```
FOR i = 1 to n
   integer Xtmp;
   Xtmp = A[i] * 3;
   B[i] = Xtmp;
```

- Eliminate by expanding into an array

```
FOR i = 1 to n
   Xtmp[i] = A[i] * 3;
   B[i] = Xtmp[i];
```

Privatization

- Need a final assignment to maintain the correct value after the loop nest

- Eliminate by assigning in local context

```
FOR i = 1 to n
   integer Xtmp;
   Xtmp = A[i] * 3;
   B[i] = Xtmp;
  if(i == n) X = Xtmp
```

- Eliminate by expanding into an array

```
FOR i = 1 to n
   Xtmp[i] = A[i] * 3;
   B[i] = Xtmp[i];
X = Xtmp[n];
```

Another Example

- How about loop-carried true dependences?
- Example

```
FOR i = 1 to n
   X = X + A[i];
```

- Is this loop parallelizable?

Reduction Recognition

- Reduction Analysis:
  - Only associative operations
  - The result is never used within the loop

- Transformation

```
Integer Xtmp[NUMPROC];
Barrier();
FOR i = myPid*Iters to MIN((myPid+1)*Iters, n)
     Xtmp[myPid] = Xtmp[myPid] + A[i];
Barrier();
If(myPid == 0) {
   FOR p = 0 to NUMPROC-1
     X = X + Xtmp[p];
   …
```

Induction Variables

- Example

```
FOR i = 0 to N
   A[i] = 2^i;
```

- After strength reduction

```
t = 1
FOR i = 0 to N
   A[i] = t;
   t = t*2;
```

- What happened to loop carried dependences?
- Need to do opposite of this!
  - Perform induction variable analysis
  - Rewrite IVs as a function of the loop variable

Array Privatization

- Similar to scalar privatization
- However, analysis is more complex
  - Array Data Dependence Analysis:
    Checks if two iterations access the same location
  - Array Data Flow Analysis:
    Checks if two iterations access the same value
- Transformations
  - Similar to scalar privatization
  - Private copy for each processor or expand with an additional dimension

Interprocedural Parallelization

- Function calls will make a loop unparallelizatble
  - Reduction of available parallelism
  - A lot of inner-loop parallelism
- Solutions
  - Interprocedural Analysis
  - Inlining

Interprocedural Parallelization

- Issues
  - Same function reused many times
  - Analyze a function on each trace → Possibly exponential
  - Analyze a function once → unrealizable path problem

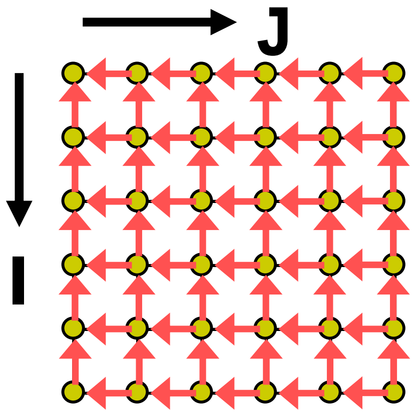
- Interprocedural Analysis
  - Need to update all the analysis
  - Complex analysis
  - Can be expensive

- Inlining
  - Works with existing analysis
  - Large code bloat → can be very expensive

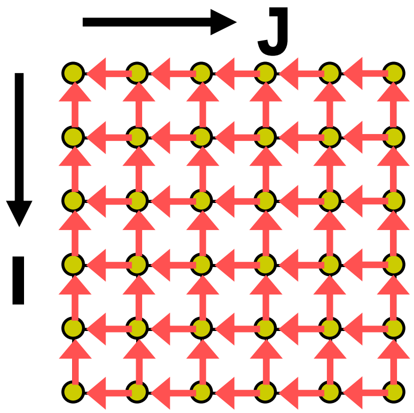
Loop Transformations

- A loop may not be parallel as is
- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
     A[i,j] = A[i,j-1] + A[i-1,j];
```

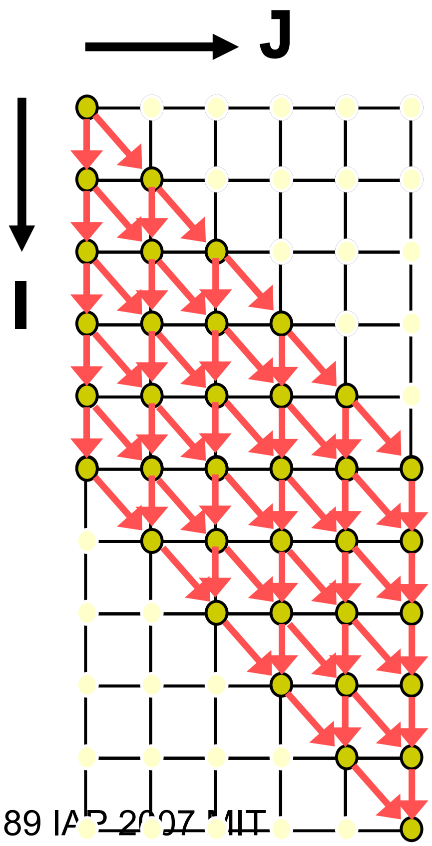


Loop Transformations



- A loop may not be parallel as is
- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
     A[i,j] = A[i,j-1] + A[i-1,j];
```



- After loop Skewing

```
FOR i = 1 to 2*N-3
   FORPAR j = max(1,i-N+2) to min(i, N-1)
     A[i-j+1,j] = A[i-j+1,j-1] + A[i-j,j];
```

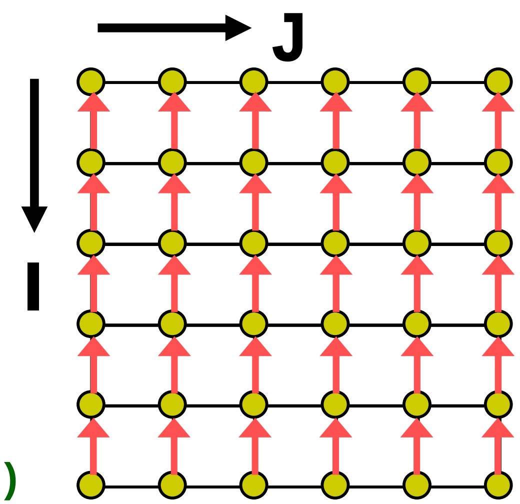
Granularity of Parallelism

- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
     A[i,j] = A[i,j] + A[i-1,j];
```

- Gets transformed into

```
FOR i = 1 to N-1
   Barrier();
   FOR j = 1+ myPid*Iters to MIN((myPid+1)*Iters, n-1)
      A[i,j] = A[i,j] + A[i-1,j];
   Barrier();
```



- Inner loop parallelism can be expensive
  - Startup and teardown overhead of parallel regions
  - Lot of synchronization
  - Can even lead to slowdowns

Granularity of Parallelism

- Inner loop parallelism can be expensive

- Solutions
  - Don’t parallelize if the amount of work within the loop is too small

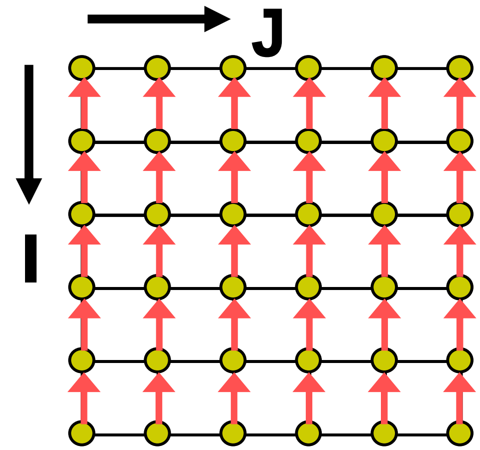
  or

  - Transform into outer-loop parallelism

Outer Loop Parallelism

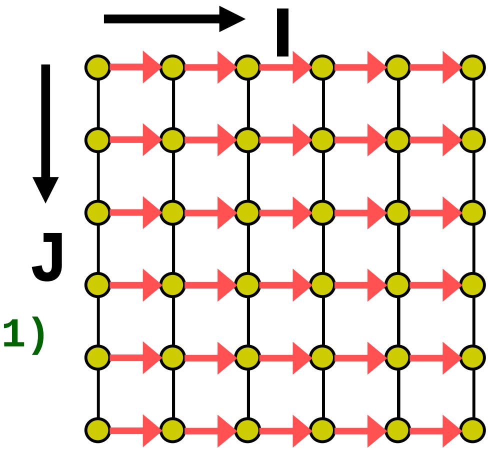
- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
      A[i,j] = A[i,j] + A[i-1,j];
```



- After Loop Transpose

```
FOR j = 1 to N-1
   FOR i = 1 to N-1
      A[i,j] = A[i,j] + A[i-1,j];
```



- Get mapped into

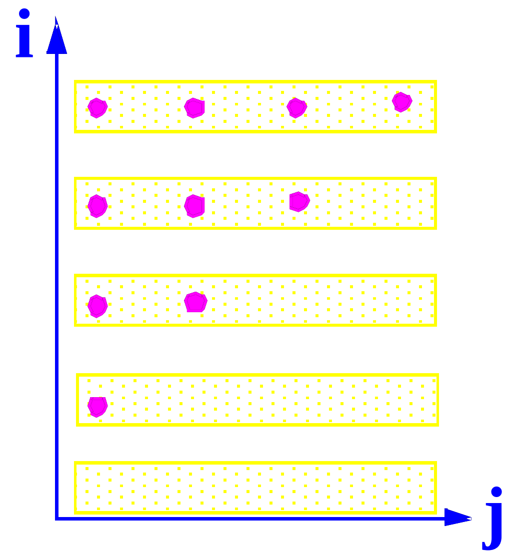
```
   Barrier();
   FOR j = 1+ myPid*Iters to MIN((myPid+1)*Iters, n-1)
      FOR i = 1 to N-1
         A[i,j] = A[i,j] + A[i-1,j];
   Barrier();
```

Outline

- Parallel Execution
- Parallelizing Compilers
- Dependence Analysis
- Increasing Parallelization Opportunities
- **Generation of Parallel Loops**
- Communication Code Generation

Generating Transformed Loop Bounds

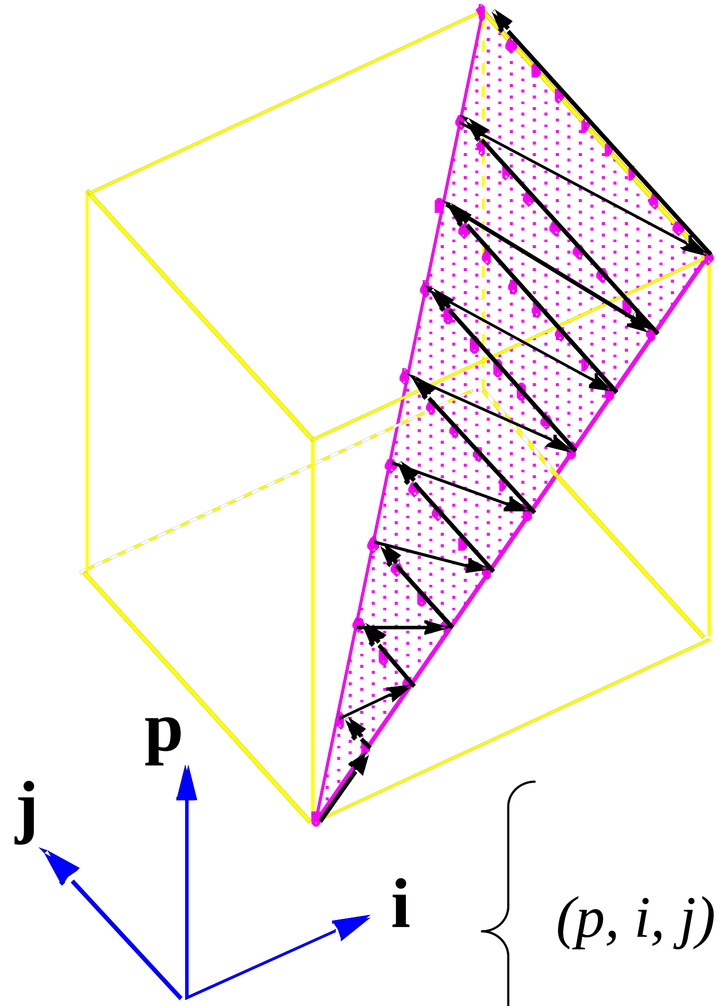
```
for i = 1 to n do
   X[i] =...
   for j = 1 to i - 1 do
        ... = X[j]
```



- Assume we want to parallelize the i loop
- What are the loop bounds?
- Use Projections of the Iteration Space
  - Fourier-Motzkin Elimination Algorithm

$$\left\{ (p, i, j) \;\middle|\; \begin{array}{c} 1 \leq i \leq n \\ 1 \leq j \leq i-1 \\ i = p \end{array} \right\}$$

Space of Iterations



$$\left\{ (p, i, j) \;\middle|\; \begin{array}{c} 1 \leq i \leq n \\ 1 \leq j \leq i\text{-}1 \\ i = p \end{array} \right\}$$

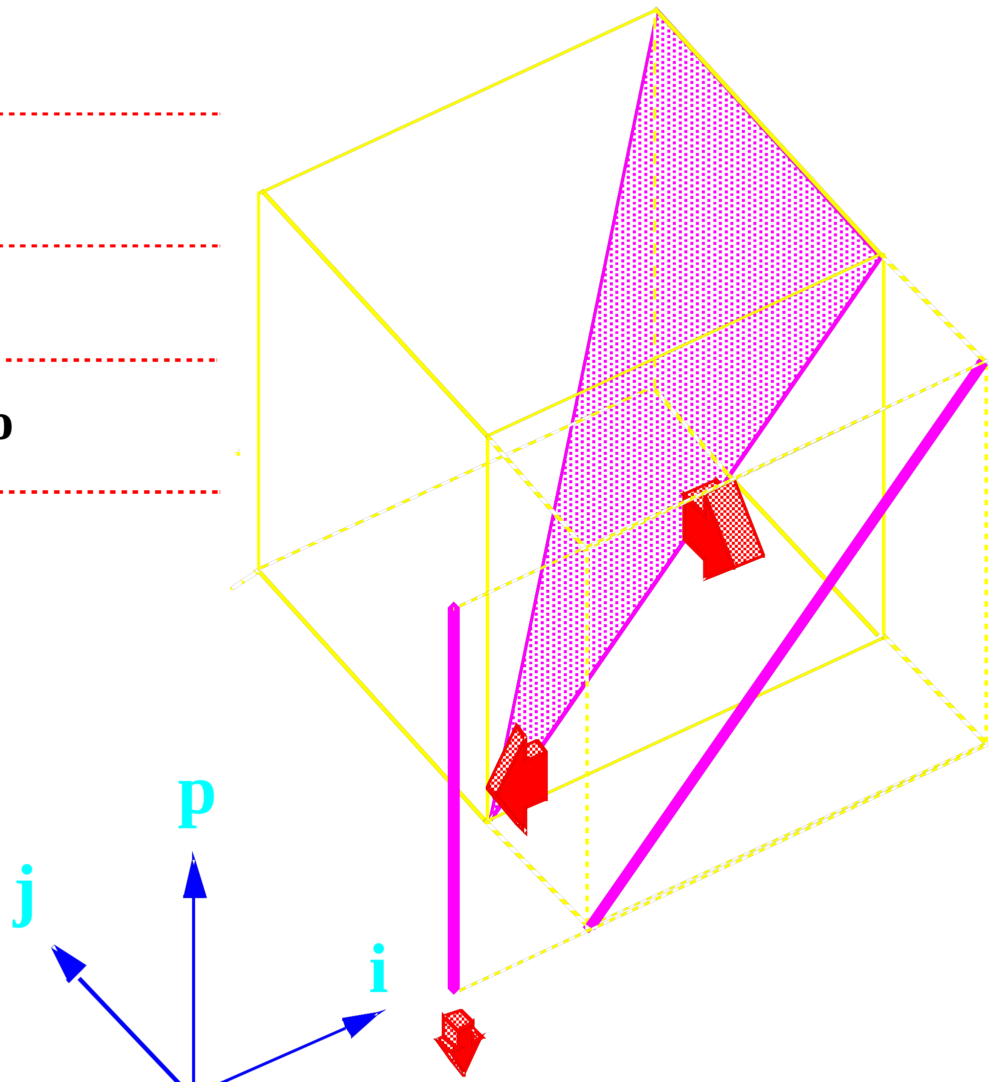
```
for p = 2 to n do
  i = p
    for j = 1 to i - 1 do
```

Projections

for p = 2 **to** n **do**

i = p

for j = 1 **to** i - 1 **do**



Projections

for p = 2 **to** n **do**

i = p

for j = 1 **to** i - 1 **do**

```
p = my_pid()
if p >= 2 and p <= n then
   i = p
   for j = 1 to i - 1 do
```

Fourier Motzkin Elimination

$1 \leq i \leq n$
$1 \leq j \leq i\text{-}1$
$i = p$

- Project i → j → p
- Find the bounds of i

$1 \leq i$
$j+1 \leq i$
$p \leq i$
$i \leq n$
$i \leq p$

i: max(1, j+1, p) to min(n, p)
i: p

- Eliminate i

$1 \leq n$
$j+1 \leq n$
$p \leq n$

$1 \leq p$
$j+1 \leq p$
$p \leq p$

$1 \leq j$

- Eliminate redundant

$p \leq n$
$1 \leq p$
$j+1 \leq p$
$1 \leq j$

- Continue onto finding bounds of j

Fourier Motzkin Elimination

$p \leq n$

$1 \leq p$

$j+1 \leq p$

$1 \leq j$

- Find the bounds of j

  $1 \leq j$

  $j \leq p - 1$

j: 1 to p – 1

- Eliminate j

$$\frac{1 \leq p - 1}{p \leq n}$$

$1 \leq p$

- Eliminate redundant

  $2 \leq p$

  $p \leq n$

- Find the bounds of p

  $2 \leq p$

  $p \leq n$

p: 2 to n

```
p = my_pid()
if p >= 2 and p <= n then
    for j = 1 to p - 1 do
        i = p
```

Outline

- Parallel Execution
- Parallelizing Compilers
- Dependence Analysis
- Increasing Parallelization Opportunities
- Generation of Parallel Loops
- **Communication Code Generation**

Communication Code Generation

- Cache Coherent Shared Memory Machine
  - Generate code for the parallel loop nest

- No Cache Coherent Shared Memory or Distributed Memory Machines
  - Generate code for the parallel loop nest
  - Identify communication
  - Generate communication code

Identify Communication

- Location Centric
  - Which locations written by processor 1 is used by processor 2?
  - Multiple writes to the same location, which one is used?
  - Data Dependence Analysis

- Value Centric
  - Who did the last write on the location read?
    - Same processor → just read the local copy
    - Different processor → get the value from the writer
    - No one → Get the value from the original array

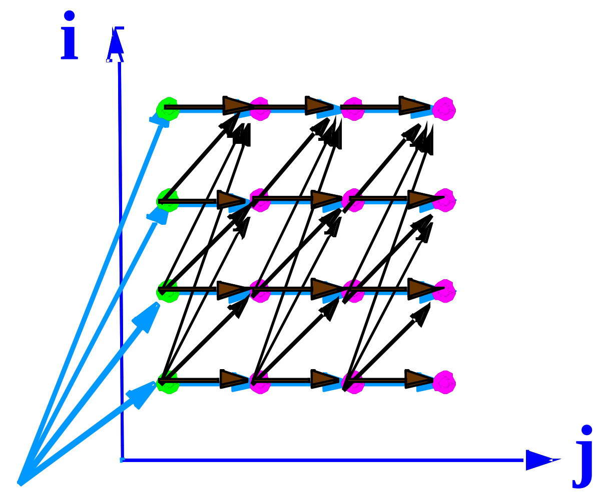
Last Write Trees (LWT)

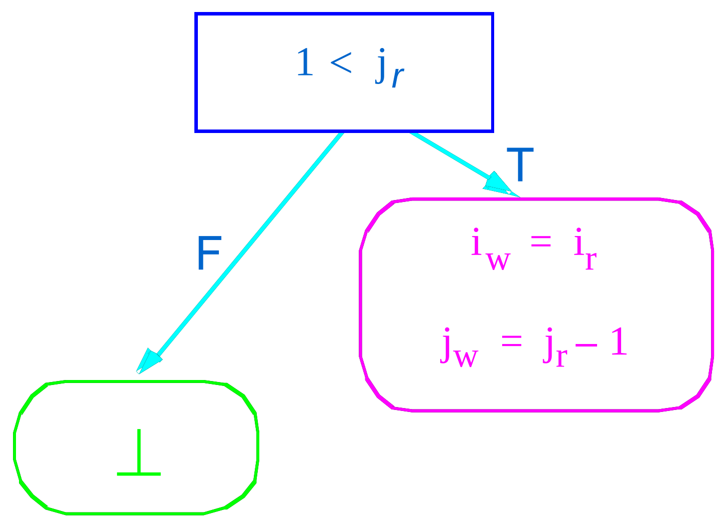
- Input: Read access and write access(es)

```
for i = 1 to n do
  for j = 1 to n do
    A[j] = …
    … = X[j-1]
```

- Output: a function mapping each read iteration to a write creating that value

Location Centric Dependences
Value Centric Dependences





The Combined Space

$$\left\{ \begin{pmatrix} p_{recv} \\ i_{recv} \\ j_{recv} \\ p_{send} \\ i_{send} \end{pmatrix} \right|$$

the receive iterations……… $1 \leq i_{recv} \leq n$

$0 \leq j_{recv} \leq i_{recv} - 1$

the last-write relation…………… $i_{send} = i_{recv}$

computation decomposition for:

receive iterations………… $P_{recv} = i_{recv}$

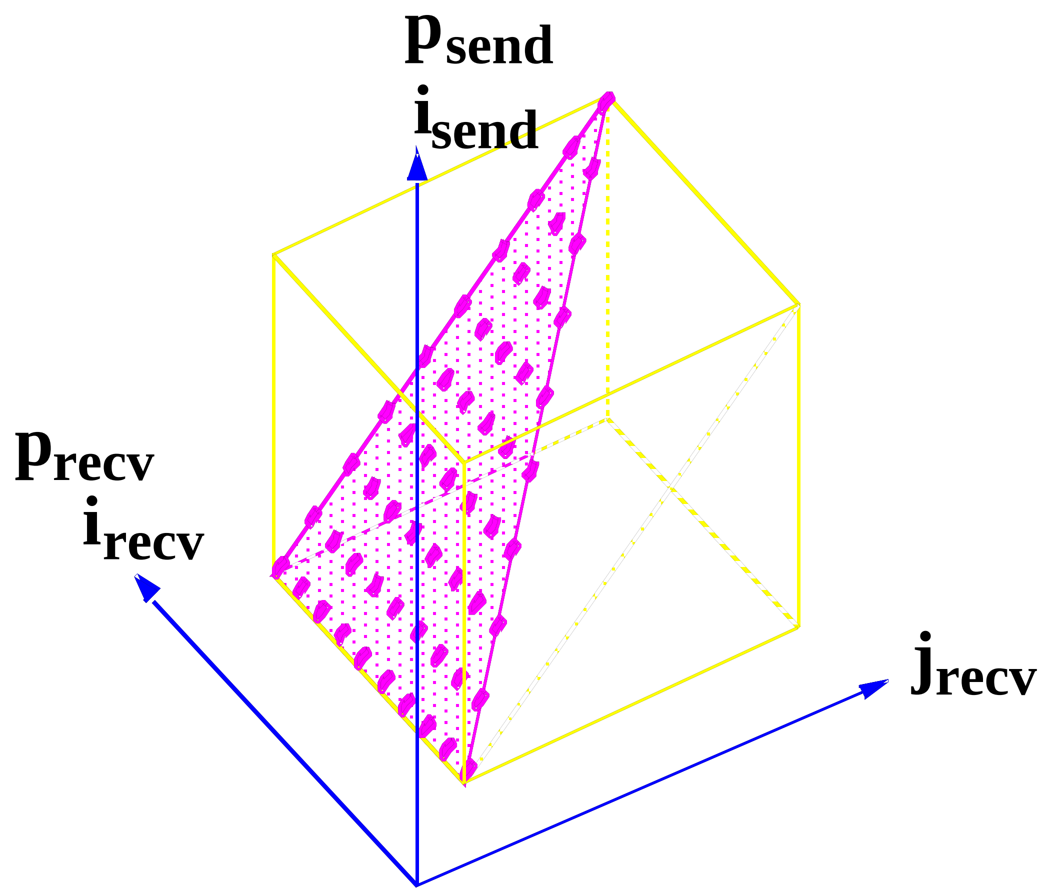
send iterations……………. $P_{send} = i_{send}$

Non-local communication…….. $P_{recv} \neq P_{send}$ $\}$

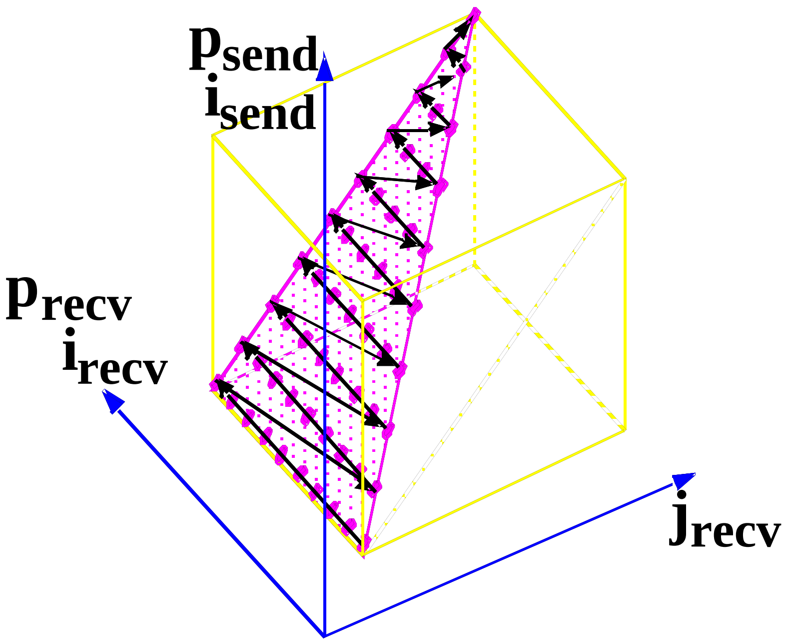
Communication Space

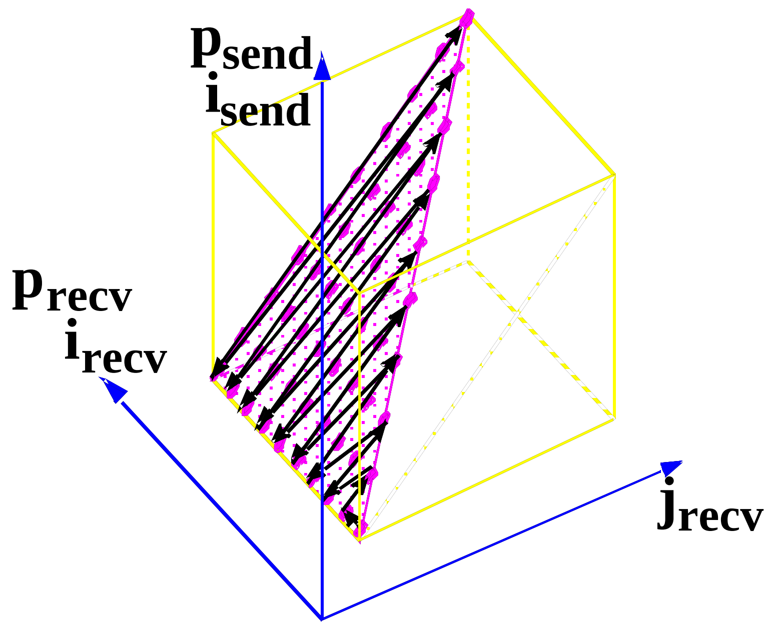
```
for i = 1 to n do
    for j = 1 to n do
    A[j] = …
    … = X[j-1]
```

$$\left\{ \begin{array}{c} 1 \le i_{recv} \le n \\ 0 \le j_{recv} \le i_{recv} - 1 \\ i_{send} = i_{recv} \\ P_{recv} = i_{recv} \\ P_{send} = i_{send} \\ P_{recv} \ne P_{send} \end{array} \right\}$$



Communication Loop Nests





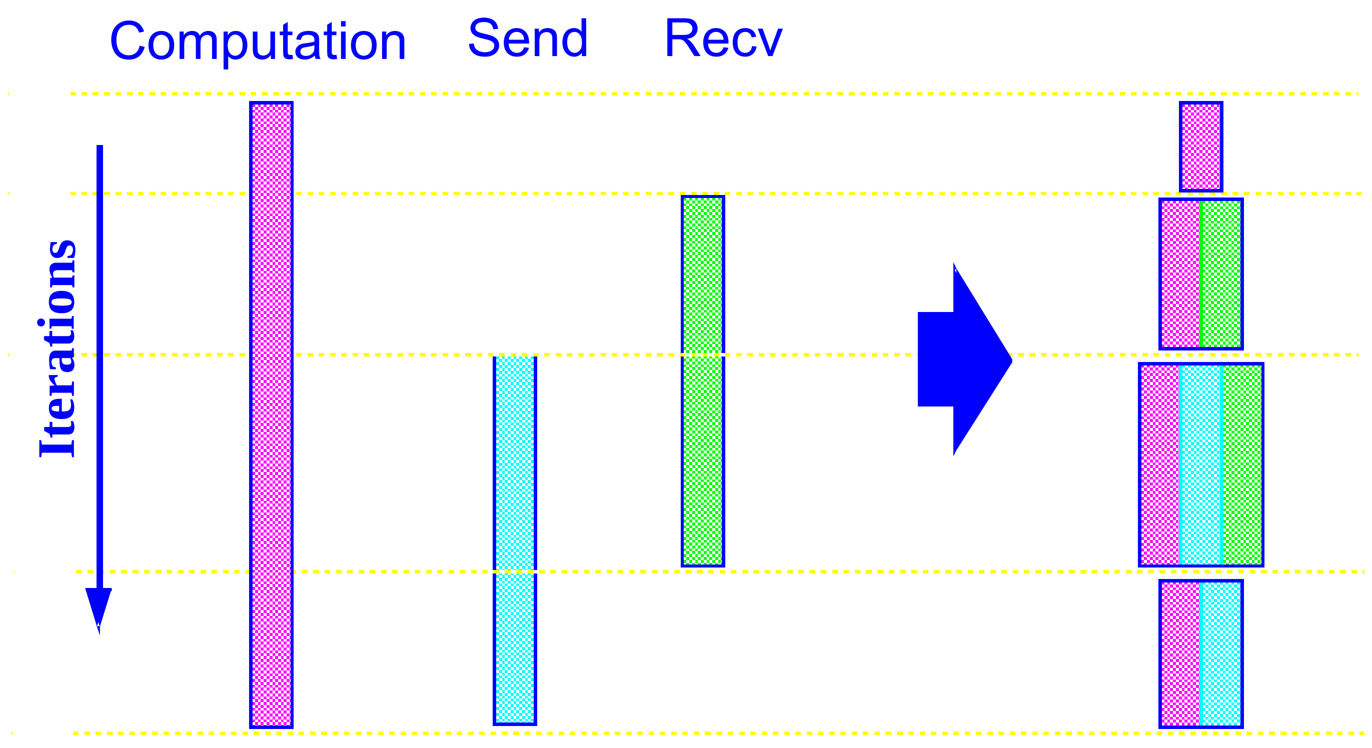
Send Loop Nest

for p_{send} = 1 **to** n - 1 **do**
  $i_{send} = p_{send}$
    for $p_{recv} = i_{send} + 1$ **to** n **do**
      $i_{recv} = p_{recv}$
      $j_{recv} = i_{send}$
      send $X[i_{send}]$ to
        iteration (i_{recv}, j_{recv}) in
        processor p_{recv}

Receive Loop Nest

for p_{recv} = 2 **to** n **do**
  $i_{recv} = p_{recv}$
  for j_{recv} = 1 **to** i_{recv} - 1 **do**
    $p_{send} = j_{recv}$
    $i_{send} = p_{send}$
    receive $X[j_{recv}]$ from
      iteration i_{send} in
      processor p_{send}

Merging Loops



Merging Loop Nests

```
if p == 1 then
   X[p] =...
   for pr = p + 1 to n do
        send X[p] to iteration (pr, p) in processor pr
if p >= 2 and p <= n - 1 then
   X[p] =...
   for pr = p + 1 to n do
        send X[p] to iteration (pr, p) in processor pr
   for j = 1 to p - 1 do
        receive X[j] from iteration (j) in processor j
        ... = X[j]
if p == n then
   X[p] =...
   for j = 1 to p - 1 do
        receive X[j] from iteration (j) in processor j
        ... = X[j]
```

Communication Optimizations

- Eliminating redundant communication
- Communication aggregation
- Multi-cast identification
- Local memory management

Summary

- Automatic parallelization of loops with arrays
  - Requires Data Dependence Analysis
  - Iteration space & data space abstraction
  - An integer programming problem
- Many optimizations that'll increase parallelism
- Transforming loop nests and communication code generation
  - Fourier-Motzkin Elimination provides a nice framework