Lecture 3. The Brain as a Network of Neurons

Reading Assignments: *

TMB2:
Section 2.3

HBTNN:
Single Cell Models (Softky and Koch)
Axonal Modeling (Koch and Bernander)
Perspective on Neuron Model Complexity (Rall)

* Unless indicated otherwise, the TMB2 material is the required reading, and the other readings supplementary.
The "basic" biological neuron

The soma and dendrites act as the input surface; the axon carries the outputs. The tips of the branches of the axon form synapses upon other neurons or upon effectors (though synapses may occur along the branches of an axon as well as the ends). The arrows indicate the direction of "typical" information flow from inputs to outputs.
For "short" cells passive propagation suffices to signal a potential change from one end to the other;

If the axon is long, this is inadequate since changes at one end would decay away almost completely before reaching the other end.

If the change in potential difference is large enough, then in a cylindrical configuration such as the axon, a pulse can actively propagate at full amplitude. The Hodgkin-Huxley Equations (1952)
Neurons and Synapses
Excitatory and Inhibitory Synapses

Dale's law states that each neuron releases a single transmitter substance.

This does not mean that the synapses made by a single neuron are either all excitatory or all inhibitory.

Modern understanding: Channels which "open" and "close" provide the mechanisms for the Hodgkin-Huxley equation, and this notion of channels extends to synaptic transmission.

The action of a synapse depends on both transmitter released presynaptically, and specialized receptors in the postsynaptic membrane.

Moreover, neurons may secrete transmitters which act as neuromodulators of the function of a circuit on some quite extended time scale (cf. TMB2 Sections 6.1 and 8.1).
**Transmembrane Ionic Transport**

*Ion channels* act as gates that allow or block the flow of specific ions into and out of the cell.
Gated Channels

A given chemical (e.g., neurotransmitter) acts as ligand and gates the opening of the channel by binding to a receptor site on the channel.
Action Potential

At rest, the inside of the cell rests at a negative potential (compared to surroundings).

Action potential consists of a brief “depolarization” (negative rest potential decreases to zero) followed by “repolarization” (inside of membrane goes back to negative rest potential), with a slight “hyperpolarization” overshoot before reaching rest.
**Action Potential and Ion Channels**

- **Initial depolarization** due to opening sodium (Na+) channels
- **Repolarization** due to opening potassium (K+) channels
- **Hyperpolarization** happens because K+ channels stay open longer than Na+ channels (and longer than necessary to exactly come back to resting potential).
Channel activations during action potential

<table>
<thead>
<tr>
<th>PHASE</th>
<th>VOLTAGE-GATED ( \text{Na}^+ ) CHANNELS</th>
<th>VOLTAGE-GATED ( \text{K}^+ ) CHANNELS</th>
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<tbody>
<tr>
<td>REST</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
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<tr>
<td>DEPOLARIZATION</td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
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<tr>
<td>PEAK</td>
<td><img src="image5" alt="Diagram" /></td>
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<td>REPOLARIZATION</td>
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<td>HYPERPOLARIZATION</td>
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A McCulloch-Pitts neuron operates on a discrete time-scale, \( t = 0, 1, 2, 3, \ldots \) with time tick equal to one refractory period.

At each time step, an input or output is

- on or off — 1 or 0, respectively.

Each connection or synapse from the output of one neuron to the input of another, has an attached weight.
Excitatory and Inhibitory Synapses

We call a synapse

- **excitatory** if $w_i > 0$, and
- **inhibitory** if $w_i < 0$.

We also associate a **threshold** $\theta$ with each neuron

A neuron fires (i.e., has value 1 on its output line) at time $t+1$ if the weighted sum of inputs at $t$ reaches or passes $\theta$:

$$y(t+1) = 1 \text{ if and only if } \sum w_i x_i(t) \geq \theta.$$
From Logical Neurons to Finite Automata


Boolean Net

Finite Automaton

Arbib and Itti: CS564 - Brain Theory and Artificial Intelligence, USC, Fall 2000. Lecture 2. Networks of Neurons
Increasing the Realism of Neuron Models

The McCulloch-Pitts neuron of 1943 is important as a basis for logical analysis of the neurally computable, and current design of some neural devices (especially when augmented by learning rules to adjust synaptic weights).

However, it is no longer considered a useful model for making contact with neurophysiological data concerning real neurons.
Leaky Integrator Neuron

The simplest "realistic" neuron model is a continuous time model based on using the firing rate (e.g., the number of spikes traversing the axon in the most recent 20 msec.) as a continuously varying measure of the cell's activity.

The state of the neuron is described by a single variable, the membrane potential.

The firing rate is approximated by a sigmoid, function of membrane potential.

\[ M(t) = \sigma(m(t)) \]
Leaky Integrator Model

\[ \tau \dot{m}(t) = -m(t) + h \]

has solution

\[ m(t) = e^{-t/\tau} m(0) + (1 - e^{-t/\tau})h \]

\[ \rightarrow h \text{ for time constant } \tau > 0. \]

We now add synaptic inputs to get the

**Leaky Integrator Model:**

\[ \tau \dot{m}(t) = -m(t) + \sum_i w_i X_i(t) + h \]

where \( X_i(t) \) is the firing rate at the \( i^{th} \) input.

Excitatory input \( (w_i > 0) \) will increase \( \dot{m}(t) \)

Inhibitory input \( (w_i < 0) \) will have the opposite effect.
Rall’s Motion Detector Model
Even at this simple level, there are alternative models.

There are inhibitory synapses which seem better described by shunting inhibition which, applied at a given point on a dendrite, serves to divide, rather than subtract from, the potential change passively propagating from more distal synapses.

The "lumped frequency" model cannot model the subtle relative timing effects crucial to our motion detector example — these might be approximated by introducing appropriate delay terms

\[ \tau m(t) = -m(t) + \sum_i w_i x_i(t - \tau_i) + h. \]
Frog Tectum: Details and Modeling

(a)

(b)

Arbib and Itti: CS564 - Brain Theory and Artificial Intelligence, USC, Fall 2000. Lecture 2. Networks of Neurons
Frog Tectum: Details and Modeling

(c) Neuron diagram showing layers and cellular structures.

(d) Tectal Column diagram illustrating neuronal connections and layers.
Many Levels of Detail in the Cerebellum
No modeling approach is automatically appropriate

Rather we seek to find the simplest model adequate to address the complexity of a given range of problems.