Solution:

Question 1

1. "Every graduate student is committed to academic honesty."
   
   Ay (Graduate(y) -> CommitedTo(y, AcademicHonesty)), or
   
   Ex AcademicHonesty(x) & Ay (Graduate(y) -> CommitedTo(y, x))
   
   Or assuming that small alphabet stands for variable:
   
   Graduate(y) -> CommitedTo(y, AcademicHonesty)
   
   Ex AcademicHonesty(x) & (Graduate(y) -> CommitedTo(y, x))

2. "Only Bill can get his car started."

   There is two different readings leading to two different answers:

   First reading: "Bill is the only individual X such that X can get Bill's car started."
   
   CanStart(Bill, car(Bill)) & Ax (CanStart(x, car(Bill)) -> x = Bill), or
   
   Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x) & Ay (CanStart(y, x) -> x = Bill))
   
   CanStart(Bill, car(Bill)) & (CanStart(x, car(Bill)) -> x = Bill), or
   
   Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x) & (CanStart(y, x) -> x = Bill))
   
   The below sentence is also complete:
   
   Ax (CanStart(x, car(Bill)) -> x = Bill)
   
   CanStart(x, car(Bill)) -> x = Bill

   Second reading: "Bill is the only individual X such that X can get X's car started."
   
   CanStart(Bill, car(Bill)) & Ax (CanStart(x, car(x)) -> x = Bill), or
   
   Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x)) & Ax ((Ey Car(y) & Owns(x, y) & CanStart(x, y)) -> x = Bill)
   
   CanStart(Bill, car(Bill)) & (CanStart(x, car(x)) -> x = Bill), or
Ex (Car(x) & Owns(Bill, x) & CanStart(Bill, x)) & ((Ey Car(y) & Owns(x, y) & CanStart(x, y)) -> x = Bill)

Ax (CanStart(x, car(x)) -> x = Bill)

CanStart(x, car(x)) -> x = Bill

3. "Things near the earth fall to the ground unless something holds them up."

Ez Ax (Near(x, Earth) & ~HoldsUp(z, x)=:=FallsTo(x,Ground)) or
They can use Ey Earth(y) and Et Ground(t) and continue the same stuff.

Ax (Near(x, Earth) -> (FallsTo(x, Ground) | Ez HoldsUp(z, x)))

4. "Dinner is available only if booked in advance for at least two persons."

Ad Ap Party(p) & Dinner(d) & BookedFor(d, p) & sizeOf(p)>=2 =>
AvailableFor(d, p)

5. "No man helps another without helping himself."

It means that if you help someone else, then you have helped yourself.

Ax Ey (Human(x) & Helps(x, y)) -> Helps(x, x)

Ax Ey Helps(x, y) -> Helps(x, x)

~ (ExEy (Human(x) & Helps(x, y)) -> ~Helps(x, x))

I hope everyone understand the same thing from this sentence.

Question 2

1) Ex Ay ~Brother(x,y)
   ~VxEy Brother(x,y)

2) Ax,y Sister(x,y) => Female(y)

3) Ax,y,f,m (Mother(x,m) && Father(x,f) && Mother(y,m) && Father(y,f)) =>
   (Sister(x,y) || Brother(x,y))

4) Ax,y, (Cousin(x,y) <=>
   (Ef1,m1,f2,m2 Father(x,f1) && Mother(x,m1) && Father(y,f2) &&
   Father(y,m2) &&
(Brother(f1,f2) || Brother(m1,f2) || Sister(m1,f2) || Sister(m1,m2))
Question 3

For this question there might be different orderings in using the sentences:

Step 1 convert to CNF

1. \( \neg \text{ice\_cream}(x) \lor \text{food}(x) \)

2. \( \neg \text{fudge}(x) \lor \text{food}(x) \)

3. \( \neg \text{food}(x) \lor \neg \text{food}(y) \lor \neg \text{cold}(x) \lor \neg \text{combine}(x, y) \lor \text{cold}(y) \)

Note that we can convert the statement

\[ \exists x \exists y \quad \text{ice\_cream}(x) \land \text{cold}(x) \land \text{fudge}(y) \land \text{combine}(x, y) \]

as follows: Start by replacing existential variables with Skolem constants \( \text{MysteryIceCream} \) for \( x \), and \( \text{MysteryFudge} \) for \( y \).

\[ \text{ice\_cream}(\text{MysteryIceCream}) \land \text{cold}(\text{MysteryIceCream}) \land \text{fudge}(\text{MysteryFudge}) \land \text{combine}(\text{MysteryIceCream}, \text{MysteryFudge}) \]

Use AND elimination to break the above statement into four new ones:

4. \( \text{ice\_cream}(\text{MysteryIceCream}) \)

5. \( \text{cold}(\text{MysteryIceCream}) \)

6. \( \text{fudge}(\text{MysteryFudge}) \)

7. \( \text{combine}(\text{MysteryIceCream}, \text{MysteryFudge}) \)

Negated Query: \( \neg \exists x \ (\neg \text{fudge}(x) \land \neg \text{cold}(x)) \)

8. \( \neg \text{fudge}(x) \lor \neg \text{cold}(x) \)

Step 2 Resolution with refutation

Combine 8 and 6 with resolution using substitution \( x/\text{MysteryFudge} \)

9. \( \neg \text{cold}(\text{MysteryFudge}) \)

Combine 1 and 4 with resolution using substitution \( x/\text{MysteryIceCream} \)

10. \( \text{food}(\text{MysteryIceCream}) \)

Combine 2 and 6 with resolution using substitution \( x/\text{MysteryFudge} \)

11. \( \text{food}(\text{MysteryFudge}) \)

Combine 11 and 3 with resolution using substitution \( y/\text{MysteryFudge} \)
12. \( \neg \text{food}(x) \vee \neg \text{cold}(x) \vee \neg \text{combine}(x, \text{MysteryFudge}) \vee \text{cold}(\text{MysteryFudge}) \)

Combine 12 and 10 with resolution using substitution \( x/\text{MysteryIceCream} \)

13. \( \neg \text{cold}(\text{MysteryIceCream}) \vee \neg \text{combine}(\text{MysteryIceCream}, \text{MysteryFudge}) \vee \text{cold}(\text{MysteryFudge}) \)

Combine 13 and 5 with resolution

14. \( \neg \text{combine}(\text{MysteryIceCream}, \text{MysteryFudge}) \vee \text{cold}(\text{MysteryFudge}) \)

Combine 14 and 7 with resolution

15. \( \text{cold}(\text{MysteryFudge}) \)

Combine 15 and 9 with resolution which results in a contradiction.

Therefore there exists cold fudge.

**Question 4**

a) \( p \rightarrow q \iff \sim p \lor q \iff (p \land \sim q) \)

b) \( p \land q \iff (\sim p \lor \sim q) \)

c) \( p \iff (p \rightarrow q) \land (q \rightarrow p) \iff (\sim (p \rightarrow q) \lor (q \rightarrow p)) \)